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Newton binomial in the generalized Cauchy problem, as exemplified by electrical systems

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ABSTRACT

Presented in the paper are direct and indirect correspondence rules between the set of real and complex coefficients of two interrelated linear differential equations of random order, each of them being able in an individual and independent way to describe uninterrupted movement of generalized, in terms of the number of freedom degrees, dynamic system with lumped parameters in the fundamental Cauchy problem, which is formulated in the first case in terms of real time functions, and in another case – in terms of their complex images, which allows directly to set one of the said forms of Cauchy problem based on the other one both in the generalized form as to the order of differential equation and in particular form under given conditions, regardless of the physical nature of the system under examination.

Keywords: dynamic system, Cauchy problem, differential equation of movement, number of freedom degrees, electrical circuit, transient process

1. INTRODUCTION

The task of analyzing transient processes in physical and technical dynamic systems with continual form of movement is overwhelmingly important and requires particular attention[1-10]. As is known, this task's mathematical interpretation is represented by the fundamental Cauchy problem [1,6,7]. Its formulation provides for construction of differential equation of dynamic system movement and determination of the totality of initial conditions with further search for particular solution.

Required for the primitive basis of the general theory of transient processes are generalized forms of Cauchy problem [6,7] being able to ensure formalization of the process the setting a particular Cauchy problem under particular conditions. The search for such generalized forms of Cauchy problem, description of their properties, detection of correlational relationships between them are in their turn the independent scientific tasks being currently important both for the theory of transient process[6,7] and for the practice thereof[2,4].

Hence, the objective of this paper lies in presentation and practical application, based on investigation of properties of electric circuits, of the rule of (operator's) correspondence between the set of real and the set of complex coefficients of two interrelated linear differential equations of random order, each of them being able in an individual and independent way to describe uninterrupted movement of generalized, in terms of the number of freedom degrees, dynamic system with lumped parameters in the fundamental Cauchy problem, which is formulated in the first case in terms of real time functions, and in another case – in terms of their complex images.

Such operator establishes direct mathematical connection between actual and complex forms of Cauchy problem, being additionally generalized by the number of the system's freedom degrees, that is why development thereof does notably strengthen the analytical capabilities of theoretical foundations of virtually all basic methods for calculation and analysis of transient processes in linear physical and technical dynamic systems with periodic form of movement[7]. Particularly – in electric circuits of sinusoid and periodic nonsinusoid currents[4].

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2. GENERALIZED CAUCHY PROBLEM IN COMPLEX FORM

Now and hereafter, we will apply the term generalized to Cauchy problem, if it is subjected to the logical action generalized at least by two attributes: the form of its expression (actual or complex) and the number of dynamic system's freedom degrees (or by the order of differential equation of its movement, which is directly related to this number).

So let us consider the linear electrical circuit of sinusoid current with one voltage source

$$u = U_m \sin\left(\omega t + \psi_u\right),$$

having a generalized topological arrangement[6,7] and a random number of freedom degrees.

In such a circuit, the transient process is described by a separate solution, for example,

$$i(t) = I_m(t)\sin\left[\omega t + \psi_i(t)\right], \tag{1}$$

of generalized Cauchy problem, which in theoretical electrical engineering is normally presented in actual form as a linear common n-th order differential equation built on the basis of Kirchhoff's laws and component interrelations with respect to instantaneous voltages or currents

$$\sum_{k=0}^{n} a_k \frac{d^k}{d\,t^k} i(t) = \sum_{s=0}^{w} b_s \frac{d^s}{d\,t^s} u(t), \tag{2}$$

complemented with n initial conditions:

$$i(0_+),\ \frac{d\,i(0_+)}{d\,t},\ldots,\frac{d^{n-1}\,i(0_+)}{d\,t^{n-1}},$$

where $i(t)$ – the transient instantaneous current in some branch of the given circuit, and $I_m(t)$ and $\psi_i(t)$ – are respectively its instantaneous amplitude and initial phase also being time functions.

At the same time, since the interrelation

$$\begin{aligned} i(t) &= \mathrm{Im}\left\{I_m(t)\cos\left[\omega t + \psi_i(t)\right] + j\,I_m(t)\sin\left[\omega t + \psi_i(t)\right]\right\} = \\ &= \mathrm{Im}\left\{I_m(t)e^{j\psi_i(t)}e^{j\omega t}\right\} = \mathrm{Im}\left\{\underline{I}_m(t)e^{j\omega t}\right\}, \end{aligned} \tag{3}$$

remains relevant for instantaneous current $i(t)$, then the Cauchy problem generalized by the said attributes may also be formulated with respect to its complex image – the instantaneous complex current

$$\underline{I}_m(t) = I_m(t)e^{j\psi_i(t)}, \tag{4}$$

based on differential equation

$$\sum_{k=0}^{n} \underline{A}_k \frac{d^k}{d\,t^k} \underline{I}_m(t) = \underline{B}_0 \underline{U}_m \tag{5}$$

and n initial conditions shaped

$$\underline{I}_m(0_+),\ \frac{d\,\underline{I}_m(0_+)}{d\,t},\ \ldots,\frac{d^{n-1}\,\underline{I}_m(0_+)}{d\,t^{n-1}},$$

which allows consider the complex time function $\underline{I}_m(t)$ as a self-sufficient mathematical object, which is able independently to describe the transient process in given electrical circuit[4].

It should be noted that theoretical electrical engineering is currently able systemically and successfully to solve the task of analyzing transient processes in electric circuits of sinusoid current virtually in all its aspects. Numerous calculation methods are proposed for this purpose, including but not limited to classical, operator and spectral methods, Duhamel integral method etc. The essence of the said methods is fundamentally presented in numerous scientific and educational literary sources, for example[1,2,4]. At the same time, as evidenced by examination of majority of the said publications, Cauchy problem is formulated in them exceptionally in an actual form in terms of instantaneous currents and voltages. Such an approach also dominates when integral transformations, particularly Fourier, Laplace or Carson integrals are used to solve the problem.

Unluckily, the possibility to form Cauchy problem with respect to functions of instantaneous complex currents of (4) type not only fails to be presented in the said source, but is not even mentioned! Apparently, this is what explains the fact that methodological support of this approach, despite the need, is now underdeveloped and insufficiently elaborated.

3. DIRECT OPERATOR OF GENERALIZED CAUCHY PROBLEM

Direct operator of generalized Cauchy problem will be the name for the law reflecting the set of real coefficients of differential equation (2) into respective set of complex coefficients of differential equation (5):

$$\mathrm{W}:\{a_0, a_1, ..., a_n; b_0, b_1, ..., b_w\} \to \{\underline{A}_0, \underline{A}_1, ..., \underline{A}_n; \underline{B}_0, \underline{B}_1 ..., \underline{B}_w\}.$$

In order to lay down this rule first of all with different *k,* let us find complex images of the first and higher derivatives of instantaneous current $i(t)$ using interrelations (1), (3) and (4):

$$i(t) \to \underline{I}_m(t)_{|k=0};$$

$$\frac{d}{dt}i(t) \to \frac{d}{dt}\underline{I}_m(t) + j\omega \underline{I}_m(t)_{|k=1};$$

$$\frac{d^2}{dt^2}i(t) \to \frac{d^2}{dt^2}\underline{I}_m(t) + j2\omega\frac{d}{dt}\underline{I}_m(t) - \omega^2\underline{I}_m(t)_{|k=2};$$

$$\frac{d^3}{dt^3}i(t) \to \frac{d^3}{dt^3}\underline{I}_m(t) + j3\omega\frac{d^2}{dt^2}\underline{I}_m(t) - 3\omega^2\frac{d}{dt}\underline{I}_m(t) - j\omega^3\underline{I}_m(t)_{|k=3}; \cdots.$$

Examination of elements of original sequence and the one created as a result of mapping allows noticing a significant regularity that makes a connection between two sets of equation coefficients (2) and (5). The essence of this regularity consists in the fact that obtained elementary mapping are individual cases of Newton binomial's general manifestation represented in a symbolic form of recording and with different k values, which belong to the set of natural numbers. The revealed regularity, which is inherent in the said transformations creates the possibility to develop their generalized formula:

$$\begin{gathered}\frac{d^k}{dt^k}i(t) \to \left(j\omega + \frac{d}{dt}\right)^k \cdot \underline{I}_m(t) = \\ = \sum_{p=0}^{k}\left(C_k^p \cdot (j\omega)^p \cdot \frac{d^{k-p}}{dt^{k-p}}\underline{I}_m(t)\right) = \sum_{p=0}^{k}\left(\frac{k!}{p!(k-p)!} \cdot (j\omega)^p \cdot \frac{d^{k-p}}{dt^{k-p}}\underline{I}_m(t)\right),\end{gathered} \tag{6}$$

where C_k^p numbers are binomial coefficients:

$$C_k^p = \frac{k!}{p!(k-p)!}.$$

In its turn, the obtained formula (6) allows establishing the rule of correspondence between the left parts of differential equations (2) and (5) the generalized Cauchy problem:

$$\sum_{k=0}^{n} a_k \frac{d^k}{d\,t^k} i(t) = \operatorname{Im}\left\{ \sum_{k=0}^{n} \left[a_k \cdot \sum_{p=0}^{k} \left(\frac{k!}{p!(k-p)!} \cdot (j\omega)^p \cdot \frac{d^{k-p}}{d\,t^{k-p}} \underline{I}_m(t) \right) \right] \cdot e^{j\omega t} \right\} =$$

$$= \operatorname{Im}\left\{ \sum_{k=0}^{n} \left[\sum_{p=0}^{n-k} \left(\frac{(k+p)!}{k!\,p!} \cdot (j\omega)^p \cdot a_{k+p} \right) \cdot \frac{d^k}{d\,t^k} \underline{I}_m(t) \right] \cdot e^{j\omega t} \right\} = \operatorname{Im}\left\{ \sum_{k=0}^{n} \underline{A}_k \cdot \frac{d^k}{d\,t^k} \underline{I}_m(t) \cdot e^{j\omega t} \right\}. \tag{7}$$

The final result in (7) interrelation is obtained due to rearrangement of all components by two sums at the same time.

The rule of correspondence between the right parts of the said equations is presented in the similar way:

$$\sum_{s=0}^{w} b_s \frac{d^s}{d\,t^s} u(t) = \operatorname{Im}\left\{ \sum_{p=0}^{w} \left[(j\omega)^p \cdot b_p \right] \cdot \underline{U}_m \cdot e^{j\omega t} \right\} = \operatorname{Im}\left\{ \underline{B}_0 \cdot \underline{U}_m \cdot e^{j\omega t} \right\}. \tag{8}$$

Formula (8) takes into account that instantaneous *complex* voltage of the source is constant under the condition of the initial problem

$$\underline{U}_m = U_m e^{j\psi_u} = \underline{const}\,,$$

whereupon its first and highest derivatives equal to zero.

The direct operator of generalized Cauchy problem is obtained directly from equations (2), (5), (7) and (8) in the form of interrelations:

$$\underline{A}_k = \sum_{p=0}^{n-k} \left[\frac{(k+p)!}{k!\,p!} \cdot (j\omega)^p \cdot a_{k+p} \right]; \;\; \underline{B}_0 = \sum_{p=0}^{w} \left[(j\omega)^p \cdot b_p \right], \tag{9}$$

where $k = 0, 1, \dots, n$, a $w \le n$.

4. INDIRECT OPERATOR OF GENERALIZED CAUCHY PROBLEM

The rule of transforming the set of coefficients of equation (5) into the set of coefficients (2):

$$\mathrm{W}^{-1} : \left\{ \underline{A}_0, \underline{A}_1, \dots, \underline{A}_n; \underline{B}_0, \underline{B}_1 \dots, \underline{B}_w \right\} \to \left\{ a_0, a_1, \dots, a_n; b_0, b_1, \dots, b_w \right\}$$

will be named, respectively, as the *indirect* operator of generalized Cauchy problem.

This operator can easily be obtained using the direct operator on the basis of the first formula (9), which allows building the system of linear algebraic equations with respect to unknown coefficients of differential equation (2). Under such approach, complex coefficients of equation (5) will serve its free terms. By solving this system, one can easily show that

$$a_k = \sum_{p=0}^{n-k} \left[\frac{(k+p)!}{k!\,p!} \cdot (-j\omega)^p \cdot \underline{A}_{k+p} \right]; \;\; b_k = \sum_{p=0}^{w-k} \left[\frac{(k+p)!}{k!\,p!} \cdot (-j\omega)^p \cdot \underline{B}_{k+p} \right]. \tag{10}$$

As regards the second interrelation in (10), it should be noted that, in case of absence of $\underline{B}_1, \dots, \underline{B}_w$ coefficients, the indirect operator is characterized by uncertainty with respect to b_k coefficients.

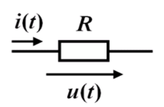
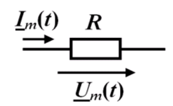
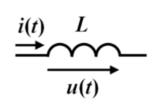
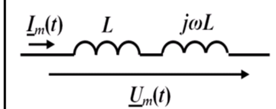
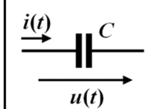
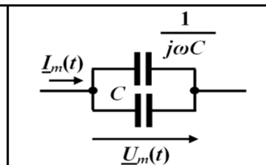
5. TRANSIENT COMPLEX CIRCUITS AND KIRCHHOFF'S LAWS IN COMPLEX TEMPORAL FORM

In theoretical and general electrical engineering, the obtained direct (9) and indirect (10) operators of generalized Cauchy problem have a wide range of practical application. In particular, they allow developing a number of methods for building a differential equation of (5) type or a system of differential equations in complex form directly on the basis of the given electric circuit diagram, and the laws and the rules, subjected to which are unknown instantaneous complex currents and voltages being the solutions of Cauchy problem in complex form and describing the course of the transient process in the given circuit. Such methods include the character-classic method for transient process calculation[4]. Its theoretical basis includes transient complex circuits and Kirchhoff's laws in a complex temporal form of presentation.

Transient compatible circuit will be the name for an object schematically and topologically structured by bipolar elements and having instantaneous complex currents in branches and by voltages in its sections mathematically connected with each other by Kirchhoff's laws and component interrelations shown in a complex temporal form of mapping.

A transient complex electric circuit can easily be built on the basis of its given pattern. The main principles of construction obtained on the basis of operators (9) and (10) obtained above are set forth in table 1, where each left column contains initial circuit elements presented term wise, and the right one – each corresponding fragment of the transient compatible circuit, which, provided that Kirchhoff's laws are complied with, is able to ensure the required component interrelation between instantaneous complex voltages and currents in a complex temporal form.

Table 1. Correspondences between the electric circuit and its transient compatible circuit.

Resistive element		Inductive element		Capacitive elemen	
					
$u = Ri$	$\underline{U}_m = R\underline{I}_m$	$u = L\frac{di}{dt}$	$\underline{U}_m = L\frac{d\underline{I}_m}{dt} + j\omega L\underline{I}_m$	$i = C\frac{du}{dt}$	$\underline{I}_m = C\frac{d\underline{U}_m}{dt} + j\omega C\underline{U}_m$

Instantaneous complex currents and voltages in a transient compatible circuit are subjected to Kirchhoff's laws in complex temporal form. These laws serve the basis for forming a system of differential equations built with respect to instantaneous complex currents in branches or voltages on them, which can further be used both for construction of differential equation of (5) type and independently – for deeper and more specific investigation of transient processes in an electrical circuit.

Let us formulate the said laws.

The first Kirchhoff's law in complex temporal form states that the algebraic total of instantaneous complex currents of the branches that concur at a node of a transient compatible circuit at any point of time, equals to zero.

The second Kirchhoff's law in complex temporal form in its turn established that the algebraic total of instantaneous complex voltages at individual elements of a random close circuit of a transient compatible circuit will at all times equal to zero. The methodology of building the equation system according to Kirchhoff's laws in complex temporal form does not differ from the known one.

6. FORMING THE CAUCHY PROBLEM IN COMPLEX FORM, AS EXEMPLIFIED BY ELECTRIC CIRCUIT

a) Let us demonstrate the immediate practical application of the obtained direct operator (9) of generalized Cauchy problem, as exemplified by a dynamic system of electro-technical origin. For this purpose, we can use the linear 2nd order (n = 2) electrical circuit, the diagram of which is shown in Fig. 1a.

As a rule, the differential equation of Cauchy problem in actual form is directly or indirectly (depending on the method) formed on the basis of Kirchhoff's laws and component interrelations, which subsequently are rewritten with respect to the unknown instantaneous voltage or current.

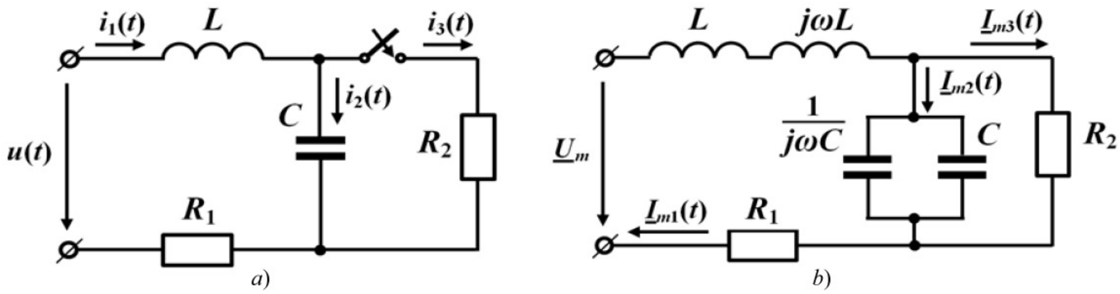


Figure 1. 2nd order electrical circuit and its transient compatible circuit.

For the given circuit, the differential equation built with respect to instantaneous current $i_1(t)$ will be presented as a common 2nd order differential equation:

$$a_2 \frac{d^2}{dt^2} i_1(t) + a_1 \frac{d}{dt} i_1(t) + a_0 i_1(t) = b_1 \frac{d}{dt} u(t) + u(t), \tag{11}$$

with coefficients:

$$\left.\begin{array}{l} a_2 = LCR_2;\ a_1 = L + CR_1R_2;\ a_0 = R_1 + R_2; \\ b_2 = 0;\ b_1 = CR_2;\ b_0 = 1. \end{array}\right\}$$

The operators of generalized Cauchy problem obtained above and demonstrating the rule of correspondence between its actual and complex forms create the opportunity for a direct transition from the actual form of type (2) equation to the complex form equation of (5) form.

Since in our case the circuit order $n = 2$, the differential equation in complex form, according to formula (5), looks as follows:

$$\underline{A}_2 \frac{d^2}{dt^2} \underline{I}_{m_1}(t) + \underline{A}_1 \frac{d}{dt} \underline{I}_{m_1}(t) + \underline{A}_0 \underline{I}_{m_1}(t) = \underline{B}_0 \underline{U}_m . \tag{13}$$

With this equation's (13) coefficients to be immediately obtained using the direct operator (9) of generalized Cauchy problem on the basis of (12) interrelations:

$$\left.\begin{array}{l} \underline{A}_2 = a_2 = LCR_2;\ \underline{A}_1 = a_1 + j2\omega a_2 = L + CR_1R_2 + j2\omega LCR_2; \\ \underline{A}_0 = a_0 - \omega^2 a_2 + j\omega a_1 = R_1 + R_2 - \omega^2 LCR_2 + j\omega\left(L + CR_1R_2\right); \\ \underline{B}_0 = b_0 - \omega^2 b_2 + j\omega b_1 = 1 + j\omega CR_2. \end{array}\right\} \tag{14}$$

b) The differential equation (13) of Cauchy problem in complex form for the given circuit and the values of its coefficients (14) may also be obtained using its transient compatible circuit on the basis of Kirchhoff's laws in complex temporal form.

For this purpose, according to the rules set forth in table 1 and based on the circuit of the given electric circuit (see Fig. 1a), let us build its transient complex diagram (Fig. 1b), and after that in a common way based of Kirchhoff's laws and component interrelations (see table 1) in complex temporal form, we build the system of differential equations for the given circuit with respect to instantaneous complex currents and voltages. The said system of equations looks as follows:

$$\begin{cases} \underline{I}_{m_1}(t) - \underline{I}_{m_2}(t) - \underline{I}_{m_3}(t) = 0, \\ \underline{I}_{m_2}(t) - C\dfrac{d}{dt}\underline{U}_{m_C}(t) - j\omega C\underline{U}_{m_C}(t) = 0, \\ L\dfrac{d}{dt}\underline{I}_{m_1}(t) + (R_1 + j\omega L)\underline{I}_{m_1}(t) + \underline{U}_{m_C}(t) = \underline{U}_m, \\ R_2\underline{I}_{m_3}(t) - \underline{U}_{m_C}(t) = 0. \end{cases} \tag{15}$$

Let us remove all other solutions, except the sought $\underline{I}_{m_1}(t)$, from system (15), and thus obtain the said differential equation (13) with coefficient (14).

c) The reference conditions of Cauchy problem in the said presentation should be calculated using the compatible pre-switching circuit on the basis of independent initial conditions and switching laws represented by the complex form[4]. In our case, such conditions will be represented by two interrelations:

$$\underline{I}_{m_1}(0_+) = \underline{U}_m\left[R_1 + j\left(\omega L - \frac{1}{\omega C}\right)\right]^{-1} \text{ and } \frac{d\,\underline{I}_{m_1}(0_+)}{dt} = 0\,.$$

7. CONCLUSIONS

Based on the Newton binomial formula, the paper represents the direct and the indirect correspondence rules between the set of real and the set of complex coefficients of two interrelated linear differential equations, each of them being able individually and independently to describe an uninterrupted movement of dynamic system generalized by the number of freedom degrees with lumped parameters in the fundamental Cauchy problem, which is formulated in the first instance in terms of real time functions, and in the other one – in terms of their complex images. The obtained results allow directly to set one of such Cauchy problem's forms through another one both in the generalized form, as regards the order of the differential equation, and in particular one under given conditions.

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