SOME ASPECTS OF USING OF THE GEOGEBRA PACKAGE IN MATHEMATICS TEACHER'S ACTIVITY

Stolyarevska A.

The Eastern-Ukrainian Branch of the International Solomon University, Ukraine

Abstract

The mathematical package GeoGebra provides for both graphical and algebraic input. It includes interactive graphics, algebra and spreadsheet, free learning materials - from elementary school to university level. One of the useful built-in capabilities of GeoGebra is the logging of the solution. This provides the students with ample opportunity not only to see the result of decision, but also to trace the steps of its construction. In addition to visualization it is also possible to use an algorithmic approach, which is equivalent to the problem solving abilities. This work gives the problems of different levels of complexity from the course of analytic geometry and translates their representation into one type of computer language – the series of instructions for the package GeoGebra.

Introduction

"Mathematics is the part of our culture that has investigated algorithmic thought in the most profound and systematic way. Now mathematics has a great deal to say about algorithmic thought" (Byers W., 2007). In fact, the question "Is thought algorithmic?" could be replaced by "Can a computer do mathematics?" or "Is mathematics algorithmic?" What aspects of mathematical activity can computers duplicate? Consider the relationship between computing and mathematics, between mathematical thought and computer simulations of thought processes. Normally using mathematics to investigate some subject means creating a mathematical model, using mathematical techniques to draw out the mathematical consequences of that model, and, finally, translating the conclusions back to the original situation.

Due to its possibility of using the algorithmic approach the package GeoGebra was chosen as the assistant for the students of the computer science faculty in their learning of analytic geometry. The main idea was to detect a gradual increase in students' level (for the students, who mostly use mathematics instrumentally) from remembering and understanding till applying, analyzing, evaluating, and creating.

The practice block

The definition of locus. The set of all points that satisfy specified conditions is called the locus of the point under the conditions. For example, the locus of points with distance a from the origin is the circle $x^2 + y^2 = a^2$ with center at (0, 0) and radius a.

The posing of the problem. For given straight line g, called the directrix, and point F, called the focus, try to find the locus of points for which the distance to the point F is in a constant ratio ε to the distance from the line g . The number ε is the eccentricity. The number p is called the focal parameter, it is the distance from the focus to the curve along a line parallel to the directrix. All sorts of curves are described by the following equation for different ε and p : $(1 - e^2)x^2 + y^2 - 2\varepsilon px = p^2$. The curves are divided into three classes, depending on the value of ε : if $\varepsilon < 1$, the curve is called an ellipse, if $e = 1$ - parabola, and otherwise - hyperbola.

Ellipse. Propositions and instructions

Table 1. Instruction 1 and explanations

Proposition 1. The ellipse can be described by comparing with the circle of radius a centered at the center of the ellipse: $x^2 + y^2 = a^2$.

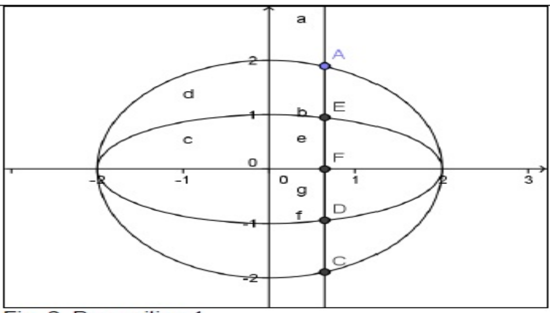

Fig. 2. Proposition 1

For each x such that $|x| \le a$, there are two points E, D on the ellipse, and two points A, C on the circle.

Conclusions:

1) The ratio of ordinates of the points E, A and D, C is equal to b/a .

2) The ellipse is obtained from the circle by compressing it to the horizontal axis, the coordinates decrease in the same ratio b/a .

3) The axes of the canonical coordinate system are the symmetry axes of the ellipse $x^2 / a^2 + y^2 / b^2 = 1$, and the origin of the coordinate system is its center of symmetry.

1		Enter a=2
2		Enter b=1
3		Enter ellipse c: $x^2 / a^2 + y^2 / b^2 = 1$
4		Enter circle d: $x^2 + y^2 = a^2$
5	A	New point A on the circle
6		Perpendicular line a to X-axis through point A
7		The intersection points for d and a are B and C (B coincides with A; hide B)
8		The intersection points for c and a are D and F
9		The intersection point for a and X-axis is F
10		Create segment b=AF, e=EF, g=FC, f=FD
11		Enter k1 = e / b; k2 = f / g
12		Save the construction.

Note. Several files are created when you save file in Geogebra:
- HTML file – this file includes the worksheet itself,
- GGB file – this file includes GeoGebra construction,
- JAR (several files) – these files include GeoGebra and make the worksheet interactive.

Supplementary problems:

1) Create point O as intersection of X-axis and Y-axis;
2) Show that the point O is the center of symmetry.

Table 2. Instruction 2

Proposition 2.
The ellipse with foci F_1 and F_2, and the major semiaxis a is the locus of points for which the sum of distances to the F_1 and F_2 is equal to $2a$.

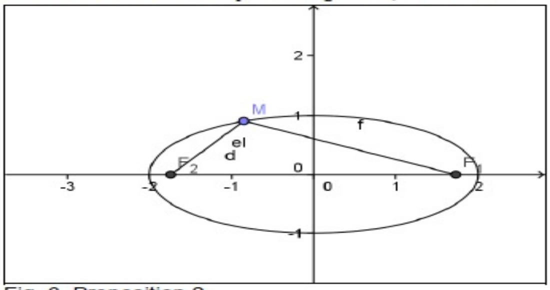

Fig. 3. Proposition 2

Proposition 3.
The distance from an arbitrary point $M(x, y)$ on the ellipse to each of the foci is a linear function of its abscissa x:
$r_1 = F_1M = a + \varepsilon x, r_2 = F_2M = a - \varepsilon x$.

1		Create two sliders a: 0..5 and b: 0..5; set a=2, b=1
2		Enter ellipse el: x² / a² + y² / b² = 1
3		Enter c= sqrt(a² - b²) ;e=c/a
4		Enter points F1=(c,0) and F2=(-c,0)
5		New point A on ellipse
6		Segment d=F2M
7		Segment f=F1M
8		Enter aa=2 a
9		Enter ff=d+f
10		Save the construction
11		Apply the drag test to check if the construction is correct.

The image on the Fig. 3 has one free element in the dynamical construction: the point M. Supplementary problem: Change the position of $M(x, y)$ and watch the value of aa and ff. Are they equal?

Table 3. The images from dynamic worksheets

Proposition 4.
The ratio of the distance between point $M(x, y)$ and the focus to the distance between point $M(x, y)$ and the directrix is equal to the eccentricity of the ellipse.

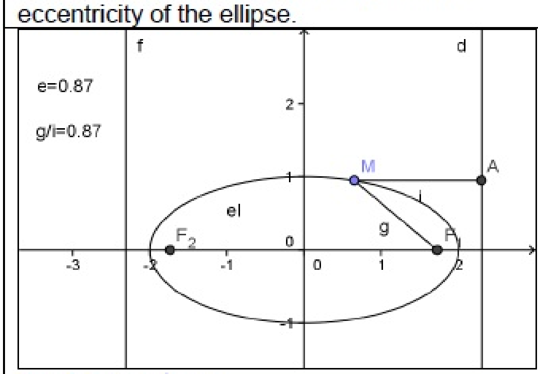

Fig. 4. Proposition 4

Proposition 5.
The tangent to the ellipse at the point $M(x, y)$ is the bisector of the angle adjacent to the angle between the segments that connect this point with the foci.

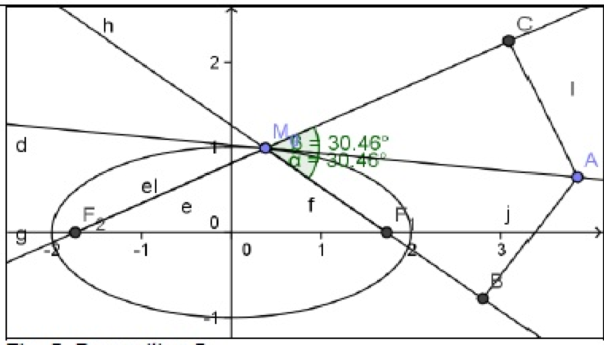

Fig. 5. Proposition 5

The images on the Fig. 4, 5 have one free element in its dynamical construction: point M on the ellipse. Supplementary problems:

1) Change the position of M(x, y) (Proposition 4) and watch the value of e and g/i.
2) Change the position of M(x, y) (Proposition 5) and watch the value of the angles AMB and AMC.

Hyperbola. Propositions and instructions

Table 4. Instruction 3

Proposition 6. The axes of the canonical coordinate system are the symmetry axes of the hyperbola $x^2/a^2 - y^2/b^2 = 1$, and the origin of the coordinate system is its center of symmetry.

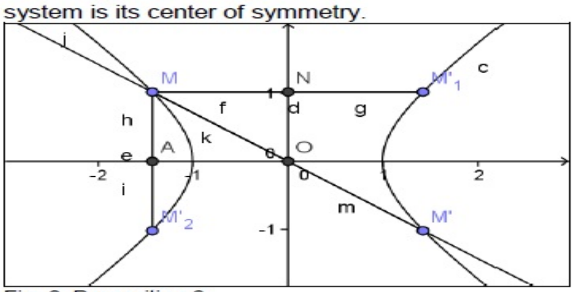

Fig. 6. Proposition 6

Conclusions:

MN=NM1, MA=AM2, and MO=OM′ for an arbitrary point $M(x, y)$, therefore the ellipse has the symmetry axes and the center of symmetry.

1	Create two sliders a: 0..5 and b: 0..5; set a=2, b=1
2	Enter hyperbola c: x² / a² - y² / b² = 1
3	New point *M* on hyperbola
4	Create O – the point of origin
5	Create line j through O and M
6	Reflect M at the Y-axis and X-axis to get its images M1 and M2.
7	Reflect M at the point O to get its image M′
8	Segments d=MM1 and e=MM2
9	Intersection point N of d and Y-axis
10	Intersection point A of e and X-axis
11	Segments MN, NM1, MA, AM2, MO, OM′

Supplementary problems:

Proposition 7. Show that if a point moves along the hyperbola so that absolute value of its abscissa increases indefinitely, then the distance from point to one of the asymptotes has zero as its limit.

Proposition 8. Show that the distances r_1, r_2 from an arbitrary point $M(x, y)$ on the hyperbola to each of the focuses dependent on its abscissa x in the following manner: $r_1 = F_1M = a - \varepsilon x r, = F_2M = a + \varepsilon x$, where $\varepsilon = c/a$ - eccentricity, a - the major semiaxis.

Table 5. The propositions of the different level of complexity

Proposition 9.

The product of the distances from the point $M(x, y)$ to the hyperbola's asymptotes is constant and is equal to $a^2b^2/(a^2+b^2)$.

Short instructions:

1) Set a and b.
2) Create hyperbola.
3) Create two asymptotes.
4) Create a point $M(x, y)$ on hyperbola.
5) Create perpendicular lines f and g to asymptotes through the point M.
6) Create segments $h = BM$ and $i = MC$.
7) Show that $h \cdot i = a^2b^2/(a^2+b^2)$

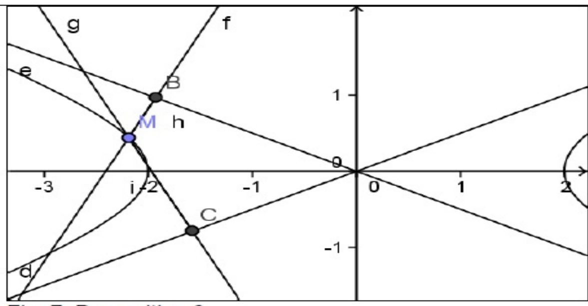

Fig. 7. Proposition 9

Proposition 10.

The hyperbola with foci F_1 and F_2 and the semiaxis a is a locus of points for which the absolute value of difference between the distances to the foci is equal to the real axis of the hyperbola.

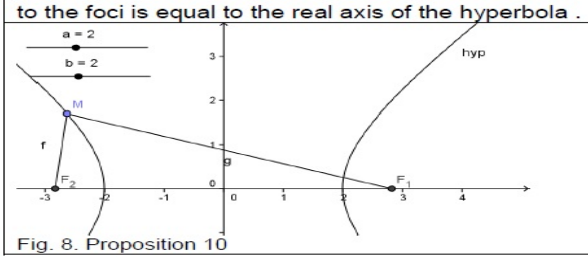

Fig. 8. Proposition 10

Proposition 11.

Tangent to the hyperbola at the point $M_0(x, y)$ is the bisector of angle between the segments, which connect this point with the foci.

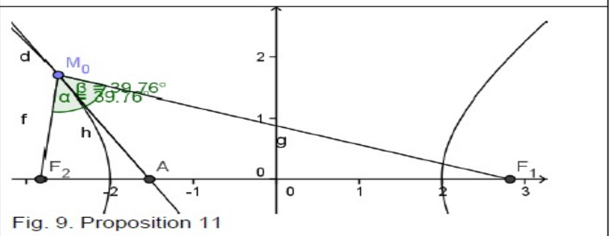

Fig. 9. Proposition 11

Supplementary problems:

1) Write short instruction for the problems in prepositions 10, 11 and perform them in GeoGebra.

2) Prove that the vertices of the hyperbola and the points of intersection of its asymptotes with directrices belong to a circle.

3) The focus of the ellipse (hyperbola or parabola) divides the chord passing through it into segments u and ν. Prove that the sum $1/\nu + 1/u$ is constant.

4) Let u and ν - the length of two perpendicular radii of the ellipse. Find the sum $1/\nu^2 + 1/u^2$.

5) Prove that the segment of the tangent concluded between the asymptotes of the hyperbola is halved in the point of contact.

Parabola. Propositions and instructions

Table 6. Instruction 4

Proposition 12. The distance from an arbitrary point $M(x, y)$ on the parabola to the focus is equal to $r = x + p/2$.

Note. Actually the square of distance from M to F is equal to $r^2 = (x - p/2)^2 + y^2 = (x - p/2)^2 + 2px = = (x + p/2)^2$. Hence, due to the fact that $x \geq 0$, it follows that $r = x + p/2$.

Proposition 13. The parabola is the locus of points equally distant from the focus and the directrix.

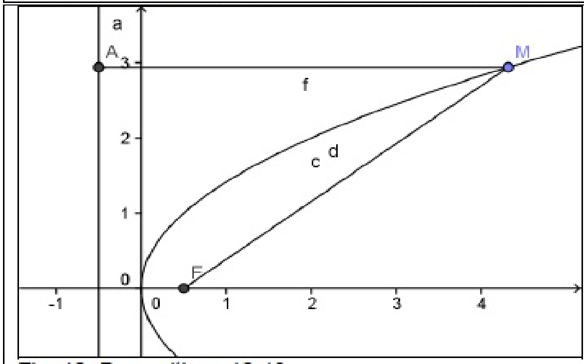

Fig. 10. Propositions 12-13

1		Create slider p: 0..5 and set p=1
2		Enter parabola c: y² - 2 p x = 0
3		Enter focus F=(p/2, 0)
4		Enter directrix x=-p/2
5		New point *M* on parabola
6		Perpendicular line to directrix through point M
7		Segment d=MF
8		Enter r=x(M)+p/2
9		Segment f=AM
10		Enter e= d/f

Supplementary problem: What is the value for the eccentricity of the parabola?

Table 7. Instruction 5

Proposition 14. The tangent to the parabola at the point $M(x, y)$ is the bisector of the angle adjacent to the angle between the segment, which connects $M(x, y)$ with focus, and the ray emerging from this point in the direction of the axis of the parabola.

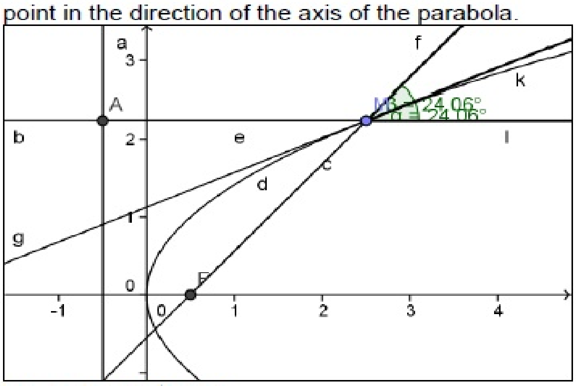

Fig. 11. Proposition 14

1		Create slider p: 0..5 and set p=1
2		Enter parabola c: y² - 2 p x = 0
3		Enter focus F=(p/2, 0)
4		Enter directrix x=-p/2
5		New point *M* on parabola
6		Tangent at M to parabola
7		Perpendicular line to directrix through point M
8		Create line through F and M

Supplementary problems:

1) Try to measure the angles (Fig. 11) and draw the conclusion.

2) Two tangents drawn to the parabola from a point located on the directrix. Prove that they are mutually perpendicular, and the segment connecting the point of contact passes through the focus.

Conclusions

With using the GeoGebra's interactive construction protocol (Hohenwarter, 2008) the students have constructed the curves as the loci of points and have solved the problems of different level of complexity. This paper is oriented on the teaching material "The second-order curves". The fourteen propositions, according to Beklemishev (2005, pp.65-88), are considered. Among the teaching materials were also used the books "Theory and problems of precalculus" (Safier, 1998), and "Geometry" (Rich, 2009).

References:

1. Beklemishev D.V. (2005) The course of analytical geometry and linear algebra. Moscow. (In Russian)
2. Byers W. (2007) How mathematicians think: using ambiguity, contradiction, and paradox to create mathematics. Princeton, New Jersey
3. Hohenwarter J., Hohenwarter M. (2008) Introduction to GeoGebra. Retrieved July 19, 2008 from http://www.geogebra.org/cms/en/help
4. Il'in V.A., Poznyak E.G. (1999) Analytical geometry, Moscow. (In Russian)
5. Rich B. (2009) Geometry includes plane, analytic, and transformational geometries. Shaum's outline of geometry. Brooklyn Technical High School, New York City.
6. Safier F. (1998) Theory and problems of Precalculus. Shaum's outline of geometry. McGRAW-HILL.