



# OPEN Novel control strategies for electric vehicle charging stations using stochastic modeling and queueing analysis

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This study presents a comprehensive analytical framework for modeling electric vehicle (EV) charging infrastructures through a stochastic queueing-theoretic approach that explicitly incorporates critical customer behavioral dynamics. The proposed model addresses key phenomena often overlooked in classical frameworks, including customer impatience (reneging), balking behavior, feedback mechanisms, and state-dependent service threshold policies, within a finite-population, multiple-server environment. These behavioral elements reflect realistic operational scenarios in which users may opt not to join extended queues, abandon the system due to excessive delays, or return for service completion based on prior dissatisfaction. The system dynamics are formulated using a continuous-time Markov chain (CTMC), and the corresponding Chapman-Kolmogorov differential equations are derived to characterize state transitions. Employing a matrix-analytic solution technique, the steady-state probability distribution is obtained, enabling the computation of multiple performance metrics such as system occupancy, server utilization, abandonment rates, and throughput. Numerical simulations validate the model's applicability and highlight intricate interdependencies among customer tolerance thresholds, service quality levels, and operational performance indicators. The findings offer valuable insights into capacity planning, congestion control, and service optimization, providing a rigorous decision-support framework for the design and management of EV charging networks under uncertain and dynamic user behavior. The study also outlines practical managerial implications and suggests directions for future research to enhance the adaptability and efficiency of smart charging infrastructures.

**Keywords** Balking behavior, Customer impatience, Electric vehicle infrastructure, Feedback mechanism, Finite-population model, Multi-server queueing, Service threshold policy, Stochastic modeling

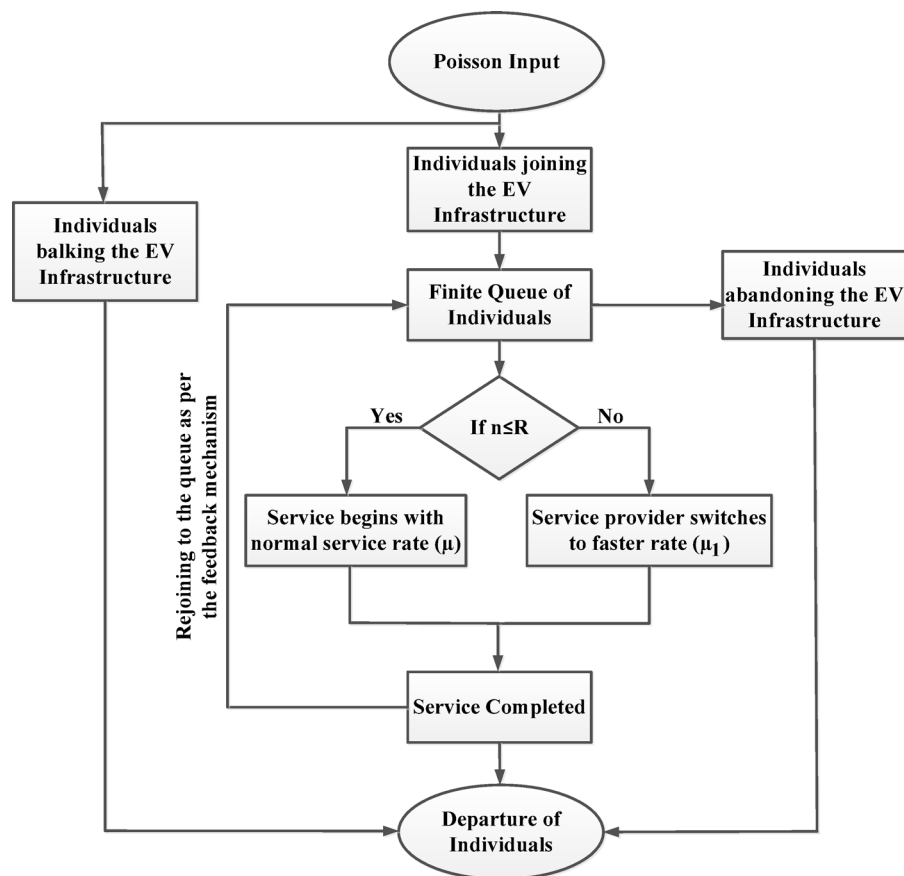
In the current era of modernization, the adoption of electric vehicles (EVs) is increasing rapidly, which consequently contributes to significant progress towards sustainable transportation. However, as more individuals prefer EVs in transportation, the increasing number of EVs generates new challenges for the existing electric power infrastructure. This transition is made significantly easier by EV charging stations, although numerous complex aspects influence operating strategies nowadays. In order to provide consistent service availability for individuals, these service stations must efficiently control service distribution, manage fluctuating demands throughout the demand interval, and ensure resource efficiency. Sometimes, the process becomes

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more complicated because of unexpected arrival patterns of individuals, varying charging requirements of various EV scenarios, and diverse customer preferences. Such an environment necessitates rigorous planning and management strategies that include both technical capabilities and customer behavior shown in Fig. 1 (The diagram presented in Fig. 1 was created using Microsoft Visio, Version 2016 (Microsoft Corporation, Redmond, WA, USA). The software is commercially licensed and available at: <https://www.microsoft.com/en-us/microsoft-365/visio/flowchart-software>). To better understand and optimize charging station operations, scholars and researchers commonly employ mathematical approaches like the concept of queues and stochastic modeling.

Although such phenomena are beneficial for examining fundamental aspects such as vehicle arrivals and charging durations, conventional models become inadequate in describing the actual behavior of customers at EV charging stations. The current research addresses these drawbacks by developing a novel queueing framework that includes multiple essential behavioral aspects namely, the impatience of individuals, service threshold policies, and the feedback mechanism of individuals (cf. Fig. 2). Flow of EVs in the EV infrastructure based on the proposed queueing framework is represented in Fig. 2 (The diagram presented in Fig. 2 was created using Microsoft Visio, Version 2016 (Microsoft Corporation, Redmond, WA, USA). The software is commercially licensed and available at: <https://www.microsoft.com/en-us/microsoft-365/visio/flowchart-software>). By incorporating these behavioral characteristics, the queueing framework provides a more accurate depiction of real-world charging station operations, contributing to enhancing the development and management of EV charging infrastructures.

Initial investigations of the research on EV charging infrastructures were aimed at developing essential foundations. The concept of EV charging infrastructure modeling emerged with Bae and Kwasinski's<sup>1</sup> pioneering spatial-temporal model for anticipating charging demand patterns. Tan et al.<sup>2</sup> modified this basic structure by adding advanced queueing network models for battery-swapping stations. Mohammed et al.<sup>3</sup> conducted a comprehensive investigation into fast charging station deployment strategies, emphasizing the dual role of these stations as both loads and storage units while exploring integration possibilities with distributed renewable energy sources. Dhaou et al.<sup>4</sup> introduced multi-priority queueing approaches that significantly transformed service efficiency frameworks, while Hosseini and MirHassani<sup>5</sup> established unique frameworks for optimal charging station location. Afterward, Panda et al.<sup>6</sup> formulated a robust residential demand-side management technique employing the binary whale optimization algorithm to assess EV impacts across various operational modes, considering EVs as loads, storage devices, and bidirectional energy resources integrated with renewable systems. Gusrialdi et al.<sup>7</sup> detailed investigation made significant advancements in distributed scheduling and



**Fig. 1.** Flow chart representing the working procedure of the EV infrastructure based on the proposed queueing framework.

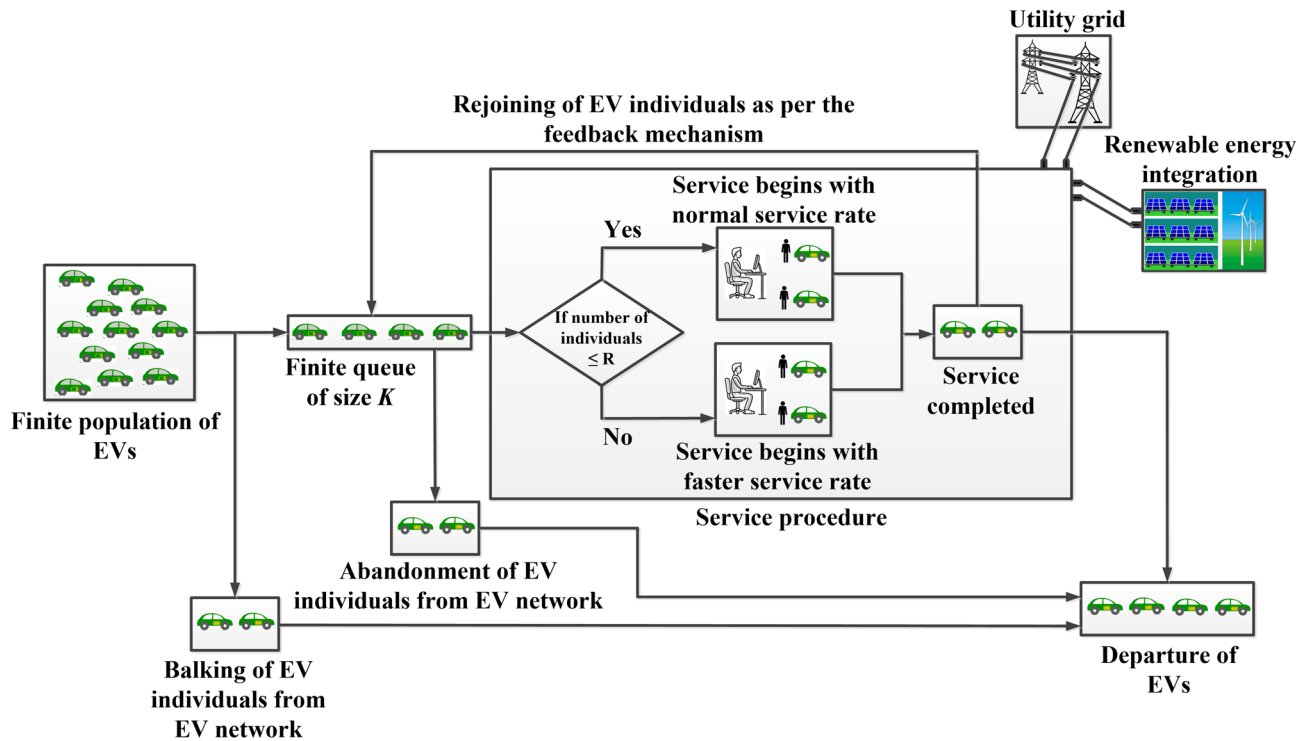


Fig. 2. Flow of EVs in the EV infrastructure based on the proposed queueing framework.

| Author(s)                          | Primary objectives                                                            | Significant outcomes                                                     |
|------------------------------------|-------------------------------------------------------------------------------|--------------------------------------------------------------------------|
| Bae & Kwasinski <sup>1</sup>       | Spatial-temporal modeling based on the charging demand.                       | Established framework for dynamic charging patterns.                     |
| Tan et al. <sup>2</sup>            | Queueing network modeling for charging stations.                              | Developed a comprehensive analytical framework for efficient operations. |
| Dhaou et al. <sup>3</sup>          | Multi-priority queueing modeling for EV charging networks.                    | Formulated a priority-based service provisioning mechanism.              |
| Hosseini & MirHassani <sup>4</sup> | Optimal location selection for charging stations.                             | Proposed strategic placement methodology.                                |
| Zengin et al. <sup>20</sup>        | Quality of service (QoS) evaluation based on multiple operational strategies. | Established performance metrics for charging networks.                   |

Table 1. Several foundational research contributions based on EV charging infrastructures.

infrastructure optimization. Saini et al.<sup>8</sup> expanded these foundational concepts by developing a two-dimensional methodology addressing both planning and operational phases for resilient electric distribution grids. Their approach appropriately incorporated stochastic modeling of EV load patterns based on geographical features and commute patterns to generate accurate probability distribution functions. Subsequent methodological advances emerged across various operational aspects, such as route selection optimization, layered queueing networks, time-dependent routing, and capacity prediction (cf. Yang et al.<sup>9</sup>, Aveklouris et al.<sup>10</sup>, Keskin et al.<sup>11</sup>, Varshosaz et al.<sup>12</sup>, Xiao et al.<sup>13</sup>, Olikar et al.<sup>14</sup>, Wang et al.<sup>15</sup>, Varshney et al.<sup>16,17</sup>, Shah et al.<sup>18</sup>). Boubaker et al.<sup>19</sup> proposed a sophisticated multi-objective framework for EV charging and discharging schedules utilizing the red-deer algorithm to optimize power flow while maintaining distribution network stability. Besides, for an in-depth investigation, a summary of some of the most significant fundamental investigations on EV infrastructures is provided in the form of Table 1.

In recent years, more advanced methodologies for modeling different operating strategies have been adopted. The field of charging management systems encountered an enormous shift with Elghitani and El-Saadany's<sup>21</sup> efficient allocation processes, followed by Bayram et al.'s<sup>22</sup> advancements in charging station architectural management. Schoenberg and Dressler<sup>23</sup>, along with Kullman et al.<sup>24</sup>, highlighted the significance and utility of waiting time management and optimal routing algorithms. Recently, Zhang et al.<sup>25</sup> developed an adaptive *N*-policy sleep scheduling approach along with the *M/M/1/C* queueing model that offers valuable insights for energy optimization in distributed systems with waiting time constraints. Recent innovations have emerged through various advanced approaches, such as queue balancing mechanisms, stochastic patterns, trading mechanisms for charging principles, and evolutionary algorithms (cf. Wang et al.<sup>26</sup>, Lyu et al.<sup>27</sup>, Hanka<sup>28</sup>, Varshney et al.<sup>29</sup>, Shekhar et al.<sup>30</sup>, Nayak et al.<sup>31</sup>). Zavvos et al.<sup>32</sup> introduced unique game-theoretic frameworks, whereas Sadhukhan et al.<sup>33</sup> advanced optimal allocation approaches for efficient charging operations. Zhang and Yang<sup>34</sup> offer an access threshold procedure for delay-sensitive (DS) tasks to control queue length and enhance load balancing across a variety of computing platforms. Kumaresan et al.<sup>35</sup> established priority-based

multi-server models, and Wang et al.<sup>36</sup> examined comprehensive vehicle-road-station interactions. Zhang et al.<sup>37</sup> introduced an active queue management (AQM) technique named as classified enhanced random early detection (CERED) and formulated a preemptive priority  $M/M/1/C$  vacation queueing framework incorporating queue management for evaluating their suggested method. The recent advancements demonstrate increasing complexity through trip-chain-based equilibrium modeling, charging/discharging methodologies, physically appropriate queueing architectures and practical application of queueing phenomena (cf. Abbasi et al.<sup>38</sup>, Tang et al.<sup>39</sup>, Said and Mouftah<sup>40</sup>, Wang and Hou<sup>41</sup>, Krivulin and Garg<sup>42</sup>, Mishra et al.<sup>43</sup>, Varshney et al.<sup>44,45</sup>), illustrating recent advancements toward integrated systems that optimize both customer experience and operational efficiency. In addition, in Table 2, some recent developments in the field of EV charging network optimization are summarized for better visualization purposes.

Moreover, Azizi et al.<sup>46</sup> contributed to the methodological advancement of optimization approaches through the energy valley optimizer (EVO), a novel metaheuristic algorithm inspired by advanced physics principles. Their approach demonstrated exceptional performance in handling complex benchmarks and real-world problems, offering promising applications for EV charging infrastructure optimization under multiple challenging constraints. Based on the extensive review of the existing literature, several significant gaps have been observed in the current research scenario of EV charging infrastructure modeling. These shortcomings highlight the requirement for advanced approaches that better reflect real-world operational challenges and customer behaviors in EV charging network circumstances.

- i). Most of the existing models address either fundamental queueing situations or isolated behavioral aspects, failing to represent the complex interrelationship between several customer and server behaviors, namely, balking, reneging, feedback mechanisms, and the threshold policies in EV charging networks.
- ii). Limited attention has been paid to characterizing the impact of frequent charging demands on service satisfaction levels and system performance, especially in finite-population scenarios with multiple homogeneous service providers.
- iii). Previous studies generally overlook the combined effects of customer impatience thresholds and service quality factors on system efficiency and resource utilization in EV charging infrastructures.
- iv). There is a lack of extensive dynamic frameworks that simultaneously address both customer departure behaviors (balking and individual abandonment) and returning patterns in the EV charging infrastructure.

To overcome the above-mentioned shortcomings, the proposed research exhibits the following significant advancements to the modeling of EV charging infrastructure. In addition to providing advantageous insights for improving EV infrastructure performance, the investigation introduces novel concepts and approaches that enhance the modeling and analysis of complex customer arrival and demand patterns.

- i). To construct an integrated queueing framework that considers all the characteristics of both customer and server behaviors, including balking, independent abandonments, feedback mechanisms, and service threshold strategies, to better represent the real-world EV charging infrastructure circumstances.
- ii). To provide a more precise capacity estimation and management of resources based on the finite-population concept that integrates multi-homogeneous servers and considers service quality measurements to improve quality-of-service (QoS) and customer satisfaction levels.
- iii). To establish numerous performance indicators that accurately reflect the interrelationship of the customer's patience constraints, service quality, and system performance, and provide significant information based on the performance of the charging infrastructure.
- iv). To provide a versatile dynamical and analytical framework that is capable of adapting to various operational circumstances while maintaining mathematical and statistical tractability, making it invaluable for both theoretical analysis and practical implementation.

Further, the organization of the current research is represented as follows: Sect. 2 describes the development of stochastic modeling via diverse assumptions and notations. The governing set of Chapman-Kolmogorov differential equations is formulated using the concepts of birth and death processes and Markov chains. Section 3 illustrates an explicit methodological approach for ascertaining the state probability distribution of the developed stochastic model, employing the integration of the matrix solution methodology. In Sect. 4, the closed-form expressions for several system performance indicators are provided to estimate the overall efficiency of EV infrastructures. For the cost analysis purpose, an expected cost function is formulated in Sect. 5.

| Author(s)                       | Primary objectives                                               | Significant outcomes                                                                                                 |
|---------------------------------|------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------|
| Kullman et al. <sup>20</sup>    | Public charging station routing strategies.                      | Developed integrated routing and charging frameworks.                                                                |
| Schoenberg et al. <sup>19</sup> | Waiting time reduction strategies.                               | Created an adaptive queue management system.                                                                         |
| Lyu et al. <sup>22</sup>        | Development of an efficient mechanism for EV charging networks.  | Established an efficient market-based framework for optimal allocation of charging resources.                        |
| Krivulin & Garg <sup>35</sup>   | Tropical modeling for the battery swapping mechanism.            | Proposed an innovative approach associated with the battery swapping management strategies.                          |
| Mishra et al. <sup>36</sup>     | EV charging stations' capacity planning and sizing optimization. | Developed a comprehensive methodology for optimal location and sizing of charging stations under demand uncertainty. |

**Table 2.** Numerous recent advancements based on charging network optimization.

Additionally, a comprehensive parametric investigation is conducted using several relevant illustrations that validate the proposed modeling and research findings, which are illustrated by numerous graphs and tables in Sect. 6. In Sect. 7, several critical managerial insights are emphasized that might benefit system designers, decision-makers, researchers, and scholars in the decision-making process. Finally, concluding remarks and several potential future perspectives, as well as an overview of prospective extensions of the suggested queueing-based stochastic modeling, are presented in Sect. 8.

## Assumptions and notations

$K$  : Capacity of the EV charging infrastructure.

$R$  : Total number of homogeneous service providers in the EV charging infrastructure.

$\lambda$  : Mean arrival rate of individuals in the EV charging infrastructure.

$\mu$  : Mean service rate of service providers during the normal working condition.

$\mu_1$  : The faster service rate utilized by service providers to reduce high congestion situations.

$\xi$  : Probability associated with the feedback mechanism of EV individuals.

$\delta$  : Probability associated with the balking behavior of EV individuals.

$\eta$  : Mean abandonment rate of EV individuals in the EV charging infrastructure.

To appropriately describe customer behavior in an EV charging infrastructure, a finite population-based queueing framework employing multiple queueing perspectives must be constructed utilizing specific assumptions and notations. These assumptions, which include reneging (the tendency for EV individuals to leave a queue), feedback mechanisms (the influence of previous experiences on subsequent behavior), and balking (the tendency for individuals to decide not to join a queue), have been demonstrated and indicate the complexity of real-world situations encountered by EV individuals. For instance, as waiting periods increase, individuals are likely to balk, which might result in decreased demand and demonstrates the crucial requirement for operational efficiency. Furthermore, the phenomenon of reneging illustrates customer discouragement, as individuals may abandon the queue in response to apparent excessive delays, thereby forcing service providers to be more prompt in service provisioning. By adding feedback mechanisms, the hypothesis emphasizes that customer experiences significantly impact future behavioral decisions, impacting patterns of charging station utilization. Collectively, these assumptions and notations allow system designers, researchers, and decision makers to develop a robust analytical framework that not only predicts individuals' behavior but also provides effective operational strategies, thereby leading to improved service provisioning and sustainability in EV charging infrastructure.

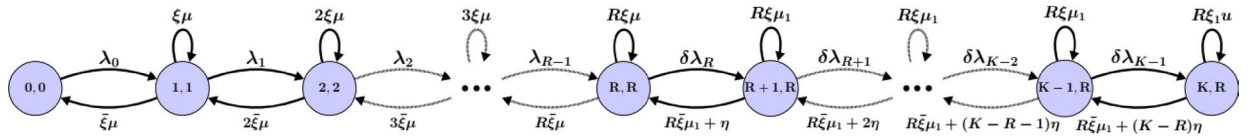
In this section, numerous assumptions and notations associated with the established queueing-based stochastic modeling are addressed using a queue-theoretic approach. Initially, the capacity of the EV charging management system is assumed to be  $K$ . It is considered that customers requiring EV charging service arrive at the charging network following an independent Poisson arrival process with an arrival rate  $\lambda$ . For the initialization of service, the First-Come-First-Served (FCFS) service discipline is utilized for each individual arrival. In the current modeling, the notion of  $R$  homogeneous service providers is taken into consideration. Further, the time-to-serve an individual customer is assumed to follow an exponential distribution with rate parameter  $\mu$ . In high-congestion situations, the service provider opts for a faster service rate in order to reduce the overload of the EV infrastructure, which is assumed to follow an exponential distribution with a rate parameter  $\mu_1$ . Another significant difference from the existing queueing models is the incorporation of feedback mechanisms in the queueing framework for each arriving customer. According to the feedback mechanism, the customer/EV individual will rejoin the EV infrastructure to complete the unsatisfactory service with probability  $\xi$  or leave the system forever with the complementary probability  $1 - \xi$ . The situation that highlights that the service provider is overloaded (or the waiting period of individuals becomes higher), activates impatience among the newly arriving customers. At that moment, due to prolonged waiting queues, the individuals may depart the EV infrastructure without getting served with probability  $\delta$  or join the EV network to get charging services with the complementary probability  $1 - \delta$ . In general, each arriving customer has a specific patience interval, upon expiration of which the customer abandons the EV infrastructure after waiting for some time. Such a type of abandonment is assumed to follow an exponential distribution with rate parameter  $\eta$ . In addition, the model determines the state-dependent arrival rate of EVs using the following equation:

$$\lambda_n = \begin{cases} (K - n) \lambda, & 0 \leq n \leq K, \\ 0, & \text{otherwise.} \end{cases}$$

Now, let us assume that  $N(t)$  represents the number of individuals in the EV charging network at time instant  $t$ , and  $J(t)$  the number of active service providers at the time  $t$ . Thus, the process  $\{(N(t), J(t)), t \geq 0\}$  constitutes an irreducible continuous-time Markov chain (CTMC) with the state-space  $\Omega = \{(n, j), n = 0, 1, \dots, K \text{ and } j = 0, 1, \dots, R\}$ . Further, combining the input and output flow from system states and employing the concept of continuous-time Markov chain (CTMC), the state transition diagram is depicted in Fig. 3.

### • Model Formulation.

In order to characterize queueing strategies effectively, the dynamical structure and specific representation of the proposed modeling must be appropriate. The primary objective of the proposed investigation is to develop a stochastic framework that precisely highlights the integration of customer impatience, service threshold policies, and the feedback mechanism employing appropriate assumptions and notations. For that purpose, the governing



**Fig. 3.** Transition state diagram associated with the proposed queueing-based EV charging network.

set of Chapman-Kolmogorov differential-difference equations (in the transient state) along with all the initial conditions is demonstrated using the transition state diagram represented in Fig. 3 as follows:

$$\frac{dP_{0,0}(t)}{dt} = -\lambda_0 P_{0,0}(t) + \bar{\xi} \mu P_{1,1}(t), \quad (1)$$

$$\frac{dP_{j,n}(t)}{dt} = -(\lambda_j + n \bar{\xi} \mu) P_{j,n}(t) + \lambda_{j-1} P_{j-1,n-1}(t) + (n+1) \bar{\xi} \mu P_{j+1,n+1}(t), \quad j = n, 1 \leq j \leq R-1, \quad (2)$$

$$\frac{dP_{R,R}(t)}{dt} = -(\delta \lambda_R + R \bar{\xi} \mu) P_{R,R}(t) + \lambda_{R-1} P_{R-1,R-1}(t) + (R \bar{\xi} \mu_1 + \eta) P_{R+1,R}(t), \quad (3)$$

$$\frac{dP_{j,R}(t)}{dt} = -(\delta \lambda_j + R \bar{\xi} \mu_1 + (j-R) \eta) P_{j,R}(t) + \delta \lambda_{j-1} P_{j-1,R}(t) + (R \bar{\xi} \mu_1 + (j-R+1) \eta) P_{j+1,R}(t), \quad \forall R+1 \leq j \leq K-1, \quad (4)$$

$$\frac{dP_{K,R}(t)}{dt} = -(R \bar{\xi} \mu_1 + (K-R) \eta) P_{K,R}(t) + \delta \lambda_{K-1} P_{K-1,R}(t). \quad (5)$$

$$\text{and } P_{0,0}(0) = 1, \quad P_{j,n}(0) = 0; \forall j, n. \quad (6)$$

In the equilibrium, i.e., as  $t \rightarrow \infty$ , the abovementioned system of Eqs. (1)–(5) can be transformed to:

$$\lambda_0 P_{0,0} = \bar{\xi} \mu P_{1,1}, \quad (7)$$

$$(\lambda_j + n \bar{\xi} \mu) P_{j,n} = \lambda_{j-1} P_{j-1,n-1} + (n+1) \bar{\xi} \mu P_{j+1,n+1}, \quad \forall j = n, \text{ and } 1 \leq j \leq R-1, \quad (8)$$

$$(\delta \lambda_R + R \bar{\xi} \mu) P_{R,R} = \lambda_{R-1} P_{R-1,R-1} + \left( R \bar{\xi} \mu_1 + \eta \right) P_{R+1,R}, \quad (9)$$

$$(\delta \lambda_j + R \bar{\xi} \mu_1 + (j-R) \eta) P_{j,R} = \delta \lambda_{j-1} P_{j-1,R} + \left( R \bar{\xi} \mu_1 + (j-R+1) \eta \right) P_{j+1,R}, \quad \forall R+1 \leq j \leq K-1, \quad (10)$$

$$(R \bar{\xi} \mu_1 + (K-R) \eta) P_{K,R} = \delta \lambda_{K-1} P_{K-1,R} \quad (11)$$

Further, in equilibrium, the normalization condition based on multiple system states can be defined as:

$$\sum_{n=0}^R P_{n,n} + \sum_{n=R+1}^K P_{n,R} = 1. \quad (12)$$

### Solution methodology

The utilization of the matrix solution technique is analyzed to depict the state probability distribution from the set of linear Eqs. (7)–(11). The system of Eqs. (7)–(11) can be transformed to the matrix equation  $\mathbf{\Pi} \mathbf{Q} = 0$ , where the probability vector  $\mathbf{\Pi}$  is a column vector of dimension  $(K+1)$  having the structure  $\mathbf{\Pi} = [P_{0,0}, P_{1,1}, P_{2,2}, \dots, P_{R-1,R-1}, P_{R,R}, P_{R+1,R}, \dots, P_{K,R}]$  and  $\mathbf{Q}$  is the tri-diagonal generator matrix having order  $(K+1) \times (K+1)$ . The structure of the generator matrix  $\mathbf{Q}$  can be represented as:

$$\mathbf{Q} = \begin{bmatrix} a_0 & b_0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ c_1 & a_1 & b_1 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & c_2 & a_2 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a_{R-1} & b_{R-1} & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & c_R & a_R & b_R & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & c_{R+1} & a_{R+1} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & a_{K-2} & b_{K-2} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & c_{K-1} & a_{K-1} & b_{K-1} \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & c_K & a_K \end{bmatrix}$$

where,

$$a_n = \begin{cases} -\left(\lambda_n + n \bar{\xi} \mu\right), & 0 \leq n \leq R-1, \\ -\left(\delta \lambda_n + R \bar{\xi} \mu_1 + (n-R)\eta\right), & R \leq n \leq K-1, \\ -\left(R \bar{\xi} \mu_1 + (K-R)\eta\right), & n = K, \\ 0, & \text{otherwise.} \end{cases}$$

$$b_n = \begin{cases} \lambda_n, & 0 \leq n \leq R-1, \\ \delta \lambda_n, & R \leq n \leq K-1, \\ 0, & \text{otherwise.} \end{cases}$$

and

$$c_n = \begin{cases} n \bar{\xi} \mu, & 1 \leq n \leq R, \\ (R \bar{\xi} \mu_1 + (n-R)\eta), & R+1 \leq n \leq K. \\ 0, & \text{otherwise.} \end{cases}$$

Now, employing the normalizing condition (12) subsequently changes the previously demonstrated framework of linear Eqs. (7)–(11) into its current form:

$$\Pi \mathbf{Q}^* = \mathbf{B}. \quad (13)$$

In the aforementioned expression,  $\mathbf{Q}^*$  denotes the matrix produced by replacing each element in the last row of the matrix  $\mathbf{Q}$  with the row vector  $\mathbf{e}_{K+1} = [1, 1, \dots, 1]_{K+1}^T$ , whereas the symbol  $\mathbf{B}$  represents the standard vector of length  $(K+1)$  of the form  $[0, 0, \dots, 0, 1]^T$ . It is evident that the matrix  $\mathbf{Q}^*$  is necessarily non-singular, and the matrix representation (13) is non-homogeneous; the final outcomes are easy to determine as  $\Pi = \mathbf{B}(\mathbf{Q}^*)^{-1}$ , which corresponds to the fundamental concepts of probability. Hence, the system of linear equations (13) possesses a solution due to the irreducibility of the generator matrix  $\mathbf{Q}$ .

Next, in order to check the existence and uniqueness of the abovementioned solution and demonstrate its matrix form, the set of differential Eqs. (1)–(5) can be rewritten as follows:

$$\frac{d\Pi_1}{dt} = \Pi_1 \mathbf{Q} \quad (14)$$

where, the vector of state probabilities  $\Pi$  is a column vector of dimension  $(K+1)$  having the structure  $\Pi_1 = [P_{0,0}(t), P_{1,1}(t), P_{2,2}(t), \dots, P_{R,R}(t), P_{R+1,R}(t), \dots, P_{K,R}(t)]$ . Following the initial conditions (cf. eq'n (6)), the initial vector  $\Pi_1(0)$  is defined as  $\Pi_1(0) = [1, 0, 0, \dots, 0]$ .

Now, let us suppose that  $\Pi_1(t) = e^{\mathbf{Q}(t-t_0)} \Pi_1(0)$ , where,

$$\Pi_1(t) = \left( \sum_{n=0}^{\infty} \frac{1}{n!} (t-t_0)^n \mathbf{Q}^n \right) \Pi_1(0) = \sum_{n=0}^{\infty} \frac{1}{n!} (t-t_0)^n \mathbf{Q}^n \Pi_1(0),$$

$$\Rightarrow \Pi_1(t_0) = e^{\mathbf{Q}(t_0-t_0)} \Pi_1(0) = \Pi_1(0).$$

Here, the function  $\Pi_1$  represents a power series with the radius of convergence  $R = \infty$  and therefore  $\Pi_1$  is differentiable, where

$$\Pi_1'(t) = \sum_{n=1}^{\infty} \frac{n}{n!} (t-t_0)^{n-1} \mathbf{Q}^n \Pi_1(0) = \sum_{n=0}^{\infty} \frac{(n+1)}{(n+1)!} (t-t_0)^n \mathbf{Q}^{n+1} \Pi_1(0),$$

$$\Rightarrow \Pi_1'(t) = \mathbf{Q} \left( \sum_{n=0}^{\infty} \frac{1}{n!} (t-t_0)^n \mathbf{Q}^n \Pi_1(0) \right) = \Pi_1(t).$$

Hence, the system of matrix Eq. (14) along with the initial vector  $\Pi_1(0)$  has a unique solution of the form  $\Pi_1(t) = e^{\mathbf{Q}(t-t_0)} \Pi_1(0)$  (cf. Deimling<sup>47</sup>), which in equilibrium state can be written as  $\Pi = \mathbf{B}(\mathbf{Q}^*)^{-1}$  along with the normalization condition (12).

### System performance measures

In the development of the electric vehicle (EV) charging infrastructures based on queueing modeling, closed-form expressions for system performance indicators are essential, especially when considering queueing nomenclatures such as feedback mechanisms, reneging, and balking. These expressions offer a systematic and effective approach for quantifying critical performance metrics, including waiting times, server utilization, and probability associated with customer abandonment, allowing for efficient investigation and comparison across diverse operational circumstances. By presenting explicit mathematical formulae, system designers and decision makers may quickly examine the impact of various design parameters on system performance and its associated cost, enhancing better decision-making. Additionally, closed-form expressions enable the integration of performance measures with an associated cost function, enabling the optimization of resource allocation and

service procedures. Thus, the formulation of these closed-form expressions becomes essential for effectively dealing with the complexities associated with modern EV charging infrastructures.

In queueing frameworks that include multiple server finite population EV infrastructures, performance metrics such as the expected number of EVs within the charging network, the expected number of active service providers, the average waiting time for individuals in the EV infrastructure, the overall throughput of the EV charging network, etc. are essential for assessing the operational efficiency of EV infrastructures. These performance indicators are formulated using advanced stochastic estimates and probabilistic modeling, providing an extensive framework for analyzing service operations. By accurately depicting the complex interrelationships of customer behavior and operational strategy, they establish prospects aimed at optimizing resource utilization and service provisioning. In real-world scenarios, these performance indicators substantially influence the anticipated total cost/revenue by offering variations in service phases, minimizing waiting times, and preventing resource constraints, hence establishing cost-effective and efficient queueing management alternatives. Further, the system performance measures utilized to analyze multiple server finite population EV charging infrastructure with queueing-based nomenclatures, namely, customer impatience, service threshold policy, and the feedback mechanism, are defined as follows:

**i) The expected number of EVs in the charging infrastructure:** The expected number of EVs in the charging infrastructure indicates the average occupancy of available charging facilities within a finite population model, providing critical insights into system utilization, facilitating effective resource management, and optimizing the customer experience.

$$E_N = 0P_{0,0} + 1P_{1,1} + 2P_{2,2} + \cdots + RP_{R,R} + (R+1)P_{R+1,R} + (R+1)P_{R+1,R} + \cdots + KP_{K,R},$$

$$\Rightarrow E_N = \sum_{j=0}^R \sum_{n=j}^R nP_{n,n} + \sum_{n=R+1}^K nP_{n,R}. \quad (15)$$

**ii) The expected number of waiting EVs in the charging infrastructure:** The expected number of waiting EVs in the charging infrastructure represents the typical number of EVs that are waiting in the queue for service. This indicator is critical for analyzing congestion levels and managing charging station capacity and efficiency.

$$E_Q = 0P_{R,R} + 1P_{R+1,R} + 2P_{R+2,R} + \cdots + (K-R)P_{K,R}, \Rightarrow E_Q = \sum_{n=R}^K (n-R)P_{n,R}. \quad (16)$$

**iii) The expected number of active service providers in the charging infrastructure:** The expected number of active service providers in the charging infrastructure indicates the average number of charging points operating at any given time. This measure is essential for understanding service capacity, minimizing waiting times, and boosting overall customer satisfaction.

$$E_B = 1P_{1,1} + 2P_{2,2} + \cdots + RP_{R,R} + RP_{R+1,R} + RP_{R+2,R} + \cdots + RP_{K,R},$$

$$\Rightarrow E_B = \sum_{j=1}^R \sum_{n=j}^R nP_{n,n} + R \sum_{n=R+1}^K P_{n,R}. \quad (17)$$

**iv) The expected number of idle servers in the charging infrastructure:** The expected number of idle servers in the charging infrastructure indicates the average number of charging facilities not in operation at a given time. This measure facilitates evaluating resource efficiency and provides recommendations based on optimal station capacity and utilization strategies.

$$E_I = RP_{0,0} + (R-1)P_{1,1} + \cdots + (R-R)P_{R,R}, \Rightarrow E_I = \sum_{j=0}^R \sum_{n=j}^R (R-n)P_{n,n}. \quad (18)$$

**v) Overall throughput of the EV charging infrastructure:** The overall throughput of the EV charging infrastructure indicates the total rate at which EVs are serviced over a certain period of time. This performance indicator is essential for evaluating system efficiency, determining capacity requirements, and providing adequate service levels for waiting customers.

$$\tau_p = \left( \bar{\xi} \mu \right) P_{1,1} + \left( 2 \bar{\xi} \mu \right) P_{2,2} + \cdots + \left( R \bar{\xi} \mu \right) P_{R,R} + \left( R \bar{\xi} \mu_1 \right) P_{R+1,R} + \left( R \bar{\xi} \mu_1 \right) P_{R+2,R} + \cdots + \left( R \bar{\xi} \mu_1 \right) P_{K,R},$$

$$\Rightarrow \tau_p = \sum_{j=1}^R \sum_{n=j}^R \left( n \bar{\xi} \mu \right) P_{n,n} + \sum_{n=R+1}^K \left( R \bar{\xi} \mu_1 \right) P_{n,R}. \quad (19)$$

**vi) The expected number of individuals abandoning the charging infrastructure:** The expected number of individuals abandoning the charging infrastructure indicates the average number of EV individuals that leave the service queue before being served. This indicator indicates customer dissatisfaction and helps in identifying challenges related to service provisioning, along with capacity optimization.

$$E_R = \eta P_{R+1,R} + 2\eta P_{R+2,R} + \cdots + (K-R)\eta P_{K,R}, \Rightarrow E_R = \sum_{n=R+1}^K (n-R)\eta P_{n,R}. \quad (20)$$

**vii) Effective arrival rate of individuals in the charging infrastructure:** The effective arrival rate of individuals in the charging infrastructure determines the actual rate at which EVs become associated with charging services, including behaviors such as balking and reneging. This indicator is essential for optimizing resource allocation and effective service provisioning.

$$\lambda_{\text{eff}} = \lambda_0 P_{0,0} + \lambda_1 P_{1,1} + \cdots + \lambda_{R-1} P_{R-1,R-1} + \lambda_R P_{R,R} + \delta \lambda_{R+1} P_{R+1,R} + \delta \lambda_{R+2} P_{R+2,R} + \cdots + \delta \lambda_{K-1} P_{K-1,R},$$

$$\Rightarrow \lambda_{\text{eff}} = \sum_{n=0}^{R-1} \lambda_n P_{n,n} + \sum_{n=R}^{K-1} \delta \lambda_n P_{n,R}. \quad (21)$$

**viii) The overall anticipated waiting time of individuals in the EV charging infrastructure:** The overall anticipated waiting time for individuals in the EV charging infrastructure determines the average duration that EV individuals spend waiting for service. This performance indicator is critical for analyzing customer experience, boosting service efficiency, and fostering overall efficiency in service operations.

$$W_S = \left( \frac{E_N}{\lambda_{\text{eff}}} \right). \quad (22)$$

**ix) The fraction of time during which servers are busy (system utilization):** The fraction of time during which servers are busy, or system utilization, represents the proportion of operational time that charging points are actively serving electric vehicles. This measure provides system operators about resource efficiency and capacity requirements, ensuring optimal performance levels.

$$U_S = \left( \frac{\lambda}{R\mu} \right). \quad (23)$$

### Cost analysis

The cost analysis and construction of the expected cost function for finite population-based EV charging infrastructure are essential for optimizing operational efficiency and sustainability. As the adoption of electric vehicles increases, it is essential to effectively manage customer behavior associated with queue dynamics, which are affected by multiple queueing characteristics, namely, balking, reneging, and feedback mechanisms at charging stations. An extensive cost analysis in terms of numerous associated costs/maintenance factors facilitates the quantification of expenditures associated with maintenance services, energy consumption, and customer system management. Furthermore, it facilitates the allocation of resources to appropriate demands, hence enhancing service reliability and customer satisfaction. The integration of specific maintenance services, including customer behavior complexities and the theory of queues, enables infrastructure operators, decision makers, and system designers to formulate proactive measures to minimize interruption and ensure continuous service availability. This hybrid strategy is essential for addressing increasing demands for electric vehicle charging while maintaining environmental sustainability and economic viability in its operations. For the formulation of the expected cost function associated with the developed queueing-based EV charging infrastructure, the following cost factors (components) are necessary.

$C_h \equiv$  Cost associated with each individual in the EV infrastructure per unit of time.

$C_b \equiv$  Cost associated with the normal busy period of the service providers in the EV infrastructure.

$C_i \equiv$  Cost associated with the idle period of the service providers in the EV infrastructure.

$C_w \equiv$  The unit cost associated with the waiting period of individuals in the EV infrastructure.

$C_m \equiv$  The unit cost associated with the maintenance services in the EV charging infrastructure.

Now, using the above-mentioned cost elements, the anticipated cost function is expressed as follows:

$$TC(\mu, R) = C_h E_N + C_b E_B + C_i E_I + C_w W_S + C_m U_S. \quad (24)$$

Therefore, the corresponding cost optimization problem can be mathematically expressed as:

$$TC^*(\mu, R) = \min_{\mu < \mu_1} TC(\mu, R). \quad (25)$$

The previously outlined cost components are considered to be constant in nature. Further, performing an analytic investigation for identifying the optimal results for the system design parameters  $\mu$ , and  $R$  becomes an extremely challenging task because of the high degree of nonlinearity and complexity of the predicted cost function. Therefore, the optimal operational strategies with the estimated cost associated with the EV charging infrastructure are calculated numerically in the current investigation.

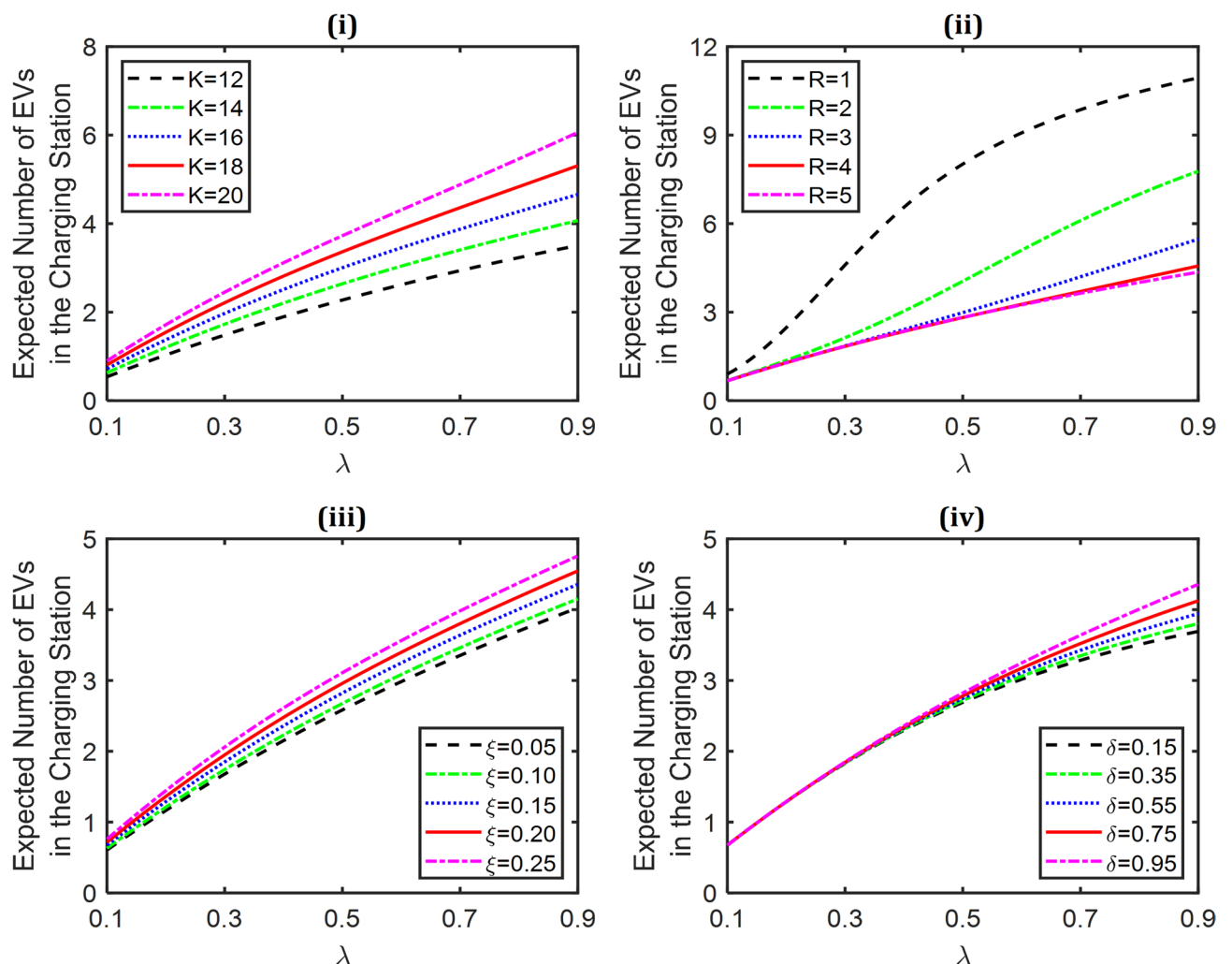
### Numerical results

In this section, several numerical illustrations are provided to validate the proposed modeling based on the queueing theoretical framework. For illustrative purposes, we have initialized the system parameters and associated cost factors as:  $K = 15$ ,  $R = 5$ ,  $\lambda = 1.5$ ,  $\mu = 2.5$ ,  $\mu_1 = 3.5$ ,  $\xi = 0.15$ ,  $\delta = 0.95$ ,  $\eta = 0.05$ ,  $C_h = 130$ ,  $C_b = 200$ ,  $C_i = 180$ ,  $C_w = 150$ ,  $C_m = 120$ . Considering hypothetical values for system parameters associated with finite population-based EV charging infrastructure, particularly those affected by queueing-based characteristics, namely, balking, reneging, and feedback mechanisms, presents significant advantages in system analysis and the architecture. By simulating multiple scenarios with assumed parameter values, system designers and decision-makers can acquire valuable insights into prospective system behavior under multiple circumstances. This approach enables the estimation

of optimal configurations for charging station layouts, resource distribution, and customer service strategies. Moreover, investigating hypothetical scenarios allows decision makers and researchers to predict customer responses to varied waiting times and service levels, fostering the development of more effective operating requirements. Such anticipation provides more substantial preparedness for peak load periods and can lead to enhanced customer satisfaction and minimize operating expenses. Overall, the investigation of potential advantages allows decision-makers to make rational decisions that match with evolving EV adoption patterns and customer expectations, resulting in a more robust and efficient charging infrastructure.

Further, the numerical results generated from the proposed modeling are presented through various line and bar graphs, demonstrating the system's behavior under different operational conditions. These parameters have been appropriately selected to represent realistic conditions at EV charging infrastructures, considering typical customer behavior, demand patterns, and operational constraints.

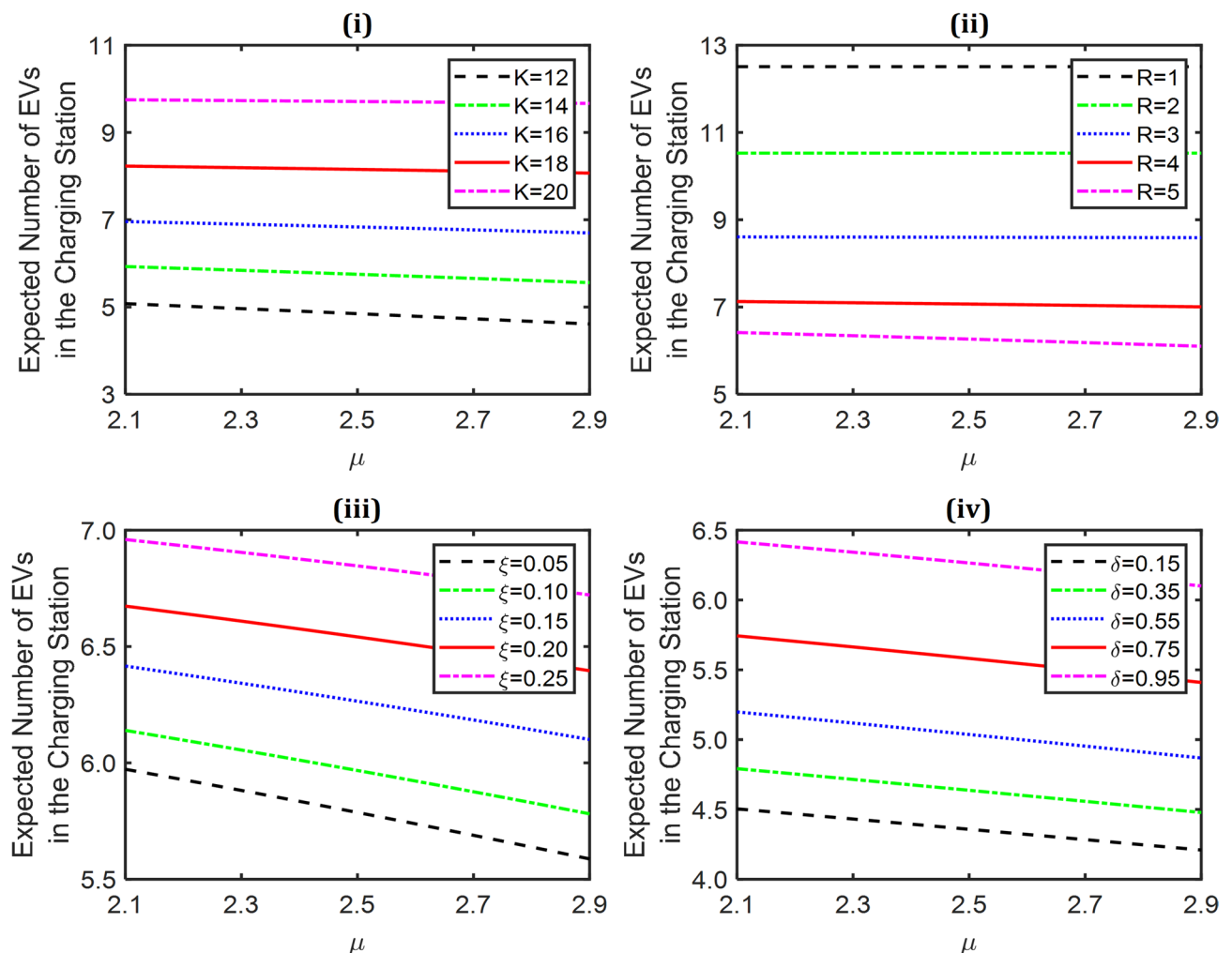
The interrelationships between the arrival rate of individuals in the EV network ( $\lambda$ ) and the expected number of EVs in the charging network ( $E_N$ ) exhibiting diverse patterns across varying system parameter values are depicted in Fig. 4. The impact of system capacity ( $K$ ) is demonstrated in Fig. 4(i), where higher system capacity ( $K$ ) values generate more acute curves, indicating better adaptation of EVs with maximum  $E_N$  reaching around 6 at  $\lambda = 0.9$ . In a similar manner, Fig. 4(ii) demonstrates that a lower proportion of service providers ( $R = 1$ ) results in much higher  $E_N$  values ( $\approx 11$  at  $\lambda = 0.9$ ), which is naturally apparent because of the fact that a lower service rate results in more congestion in the EV infrastructure. Besides, the increased number of service providers ( $R = 5$ ) ensures more significant system control. The fluctuation in the feedback probability ( $\xi$ ) in Fig. 4(iii) reveals that an increased value of  $\xi$  leads to substantially higher  $E_N$ , particularly at increased arrival rates. Figure 4(iv) depicts the impact of balking probability ( $\delta$ ) on the EV infrastructure, where curves remain essentially identical until  $\lambda \approx 0.5$ , after which higher joining rates exhibit slightly increased  $E_N$  values. These results collectively indicate that system capacity and different types of customer preferences have the most significant impact on EV charging station performance and operational characteristics.



**Fig. 4.** The impact of arrival rate ( $\lambda$ ) and other system parameters on the expected number of EVs in the charging station ( $E_N$ )

In Fig. 5, the impact of the service rate ( $\mu$ ) on the expected number of EVs in the charging network ( $E_N$ ) is presented based on the distinctive characteristics with respect to the varied values of the other system parameters. The impact of system capacity ( $K$ ) along with varied values of service rate ( $\mu$ ) is demonstrated in Fig. 5(i), where higher  $K$  values convey higher  $E_N$  values such as with respect to  $K = 20$ , the  $E_N$  value reaching around 9.5, and for  $K = 12$ , the minimum  $E_N$  is observed, which is around 5 at  $\mu = 2.1$ . It is apparent from the fact that the lower value of system capacity is less effective in increasing the  $E_N$  values. Additionally, Fig. 5(ii) demonstrates that the reduced number of service providers ( $R = 1$ ) results in much higher  $E_N$  values ( $\approx 12.5$ ), while the higher value of service providers ( $R = 5$ ) maintains the lower  $E_N$  levels ( $\approx 6.5$ ), revealing a strong inverse relationship. This is because of the fact that the improved service rate substantially decreases the expected number of individuals in the EV infrastructure and improves the satisfaction level of individuals. The fluctuation in the feedback probability ( $\xi$ ) in Fig. 5(iii) demonstrates significant variation, with  $\xi = 0.25$  achieving  $E_N \approx 7.0$  and  $\xi = 0.05$  exhibiting  $E_N \approx 6.0$  at  $\mu = 2.1$ . It is due to the fact that as more and more customers join the EV infrastructure to further fulfill their unsatisfactory service as per the feedback procedure, the  $E_N$  value considerably increases. Figure 5(iv) depicts the impact of the joining probability ( $\delta$ ), demonstrating the smallest fluctuations among all factors, ranging from  $E_N \approx 4.5$  at  $\delta = 0.15$  to  $E_N \approx 6.5$  at  $\delta = 0.95$  with respect to  $\mu = 2.1$ . All curves consistently demonstrate declining trends as  $\mu$  increases from 2.1 to 2.9, with the sharpest descending trend observed in higher system capacity values as well as the lesser impact based on the number of service providers. These results collectively suggest that system capacity and feedback constraints have the most substantial impact on EV charging station performance, while balking and abandoning behaviors demonstrate deeper implications on system behavior, with values generally converging at higher  $\mu$  values showing reduced parameter sensitivity.

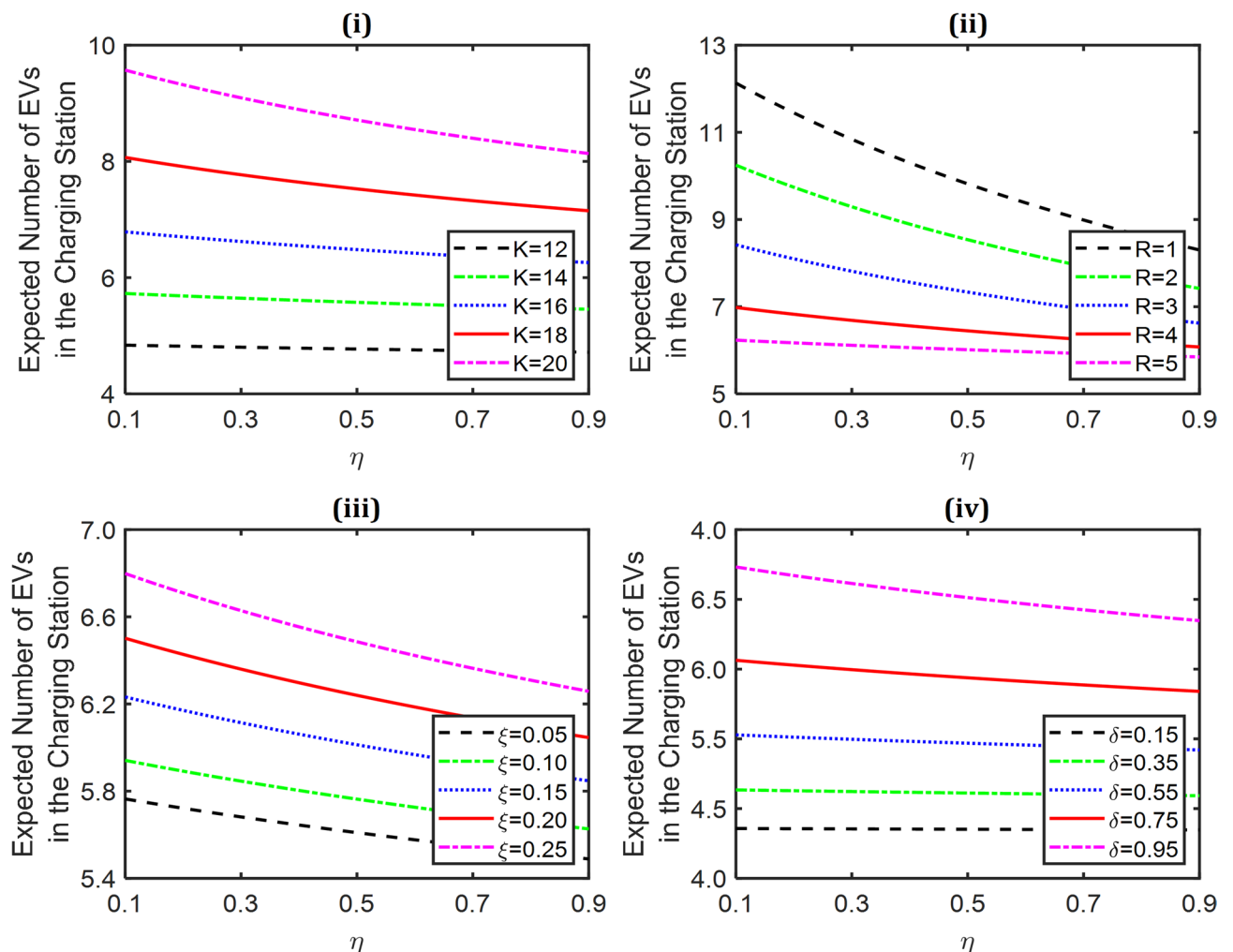
There are significant patterns in the correlation between the expected number of EVs in the charging station ( $E_N$ ) and the reneging rate ( $\eta$ ), depending on a number of system characteristics. As indicated in Fig. 6(i), larger system capacity ( $K$ ) levels, specifically  $K = 20$ , correlate with an  $E_N$  maximum of approximately 9.5 with respect to the reneging rate  $\eta = 0.1$ . Conversely, a lesser capacity ( $K = 12$ ) results in a linear pattern



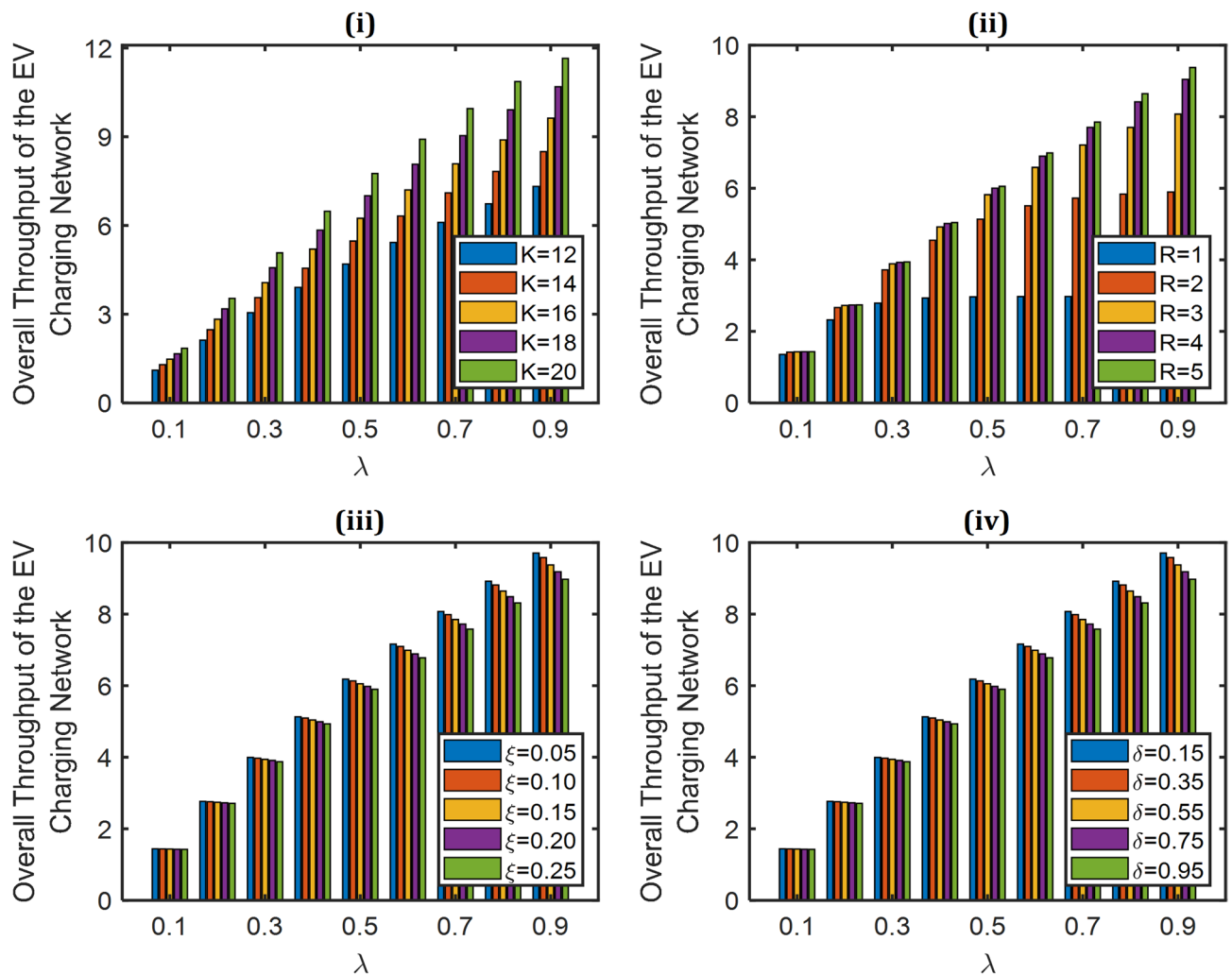
**Fig. 5.** The impact of service rate ( $\mu$ ) and other system parameters on the expected number of EVs in the charging station ( $E_N$ ).

of  $E_N$  across all  $\eta$  values. As observed in Fig. 6(ii), a lower number of service providers ( $R = 1$ ) results in much higher  $E_N$  levels ( $\approx 12$  at  $\eta = 0.1$ ), but higher  $R$  values ( $R = 5$ ) preserve a more constant  $E_N$  pattern, reaching approximately the value  $E_N = 6$ . In addition, the variation in the parameter associated with the feedback probability ( $\xi$ ) highlights substantial system sensitivity as demonstrated in Fig. 6(iii). Similarly, Fig. 6(iv) examines the parameter associated with the joining probability ( $\delta$ ), indicating the smallest variability in  $E_N$  such as for  $\delta = 0.95$ , the  $E_N$  value reaching approximately 6.7, and for  $\delta = 0.15$ , it stabilizes at  $E_N \approx 4.3$  with respect to  $\eta = 0.1$ . As the rate of abandonment  $\eta$  increases from 0.1 to 0.9,  $E_N$  continues to decrease in all patterns, with the most noticeable decrease occurring in EV infrastructures that have a substantial impact on system capacity and a lesser number of service providers.

The correlation between the arrival rate ( $\lambda$ ) and the overall throughput of the EV infrastructure ( $\tau_p$ ) reveals diverse patterns across various system parameter values. The impact of system capacity ( $K$ ) is demonstrated in Fig. 7(i), where higher  $K$  values result in increasing throughput value ( $\tau_p$ ), reaching maximum values of around 11 units at  $\lambda = 0.9$ , while lower  $K$  values maintain comparatively lesser  $\tau_p$  values around 7–8 units, which is intuitively anticipated. The influence of the increasing number of service providers ( $R$ ) is relatively higher, as seen in Fig. 7(ii), where values from  $R = 1$  to  $R = 5$  exhibit comparable increasing trends, achieving  $\tau_p$  values between 8 and 9 units at  $\lambda = 0.9$ . It is because of the fact that as the number of service providers increases in the EV infrastructure, intuitively, overall throughput will also increase. The fluctuation in the parameter value of feedback probability ( $\xi$ ) is observed in Fig. 7(iii), which demonstrates generally evenly evolving behavior across all distinct  $\lambda$  values. Similarly, Fig. 7(iv) depicts the impact of the parameter associated with the joining probability ( $\delta$ ), showing essentially comparable patterns across multiple  $\delta$  values, where  $\tau_p$  increases steadily as  $\lambda$  increases, reaching peak values of around 9 units at  $\lambda = 0.9$ . All of the illustrations display exponential growth patterns as  $\lambda$  increases from 0.1 to 0.9, with the most pronounced effects observed in system capacity fluctuations. These results reveal that system capacity ( $K$ ) has the most significant impact on the overall throughput of the EV infrastructure, while feedback probability, the parameter correlated with the impatience behavior of individuals, has comparatively lesser effects on system performance. In a similar fashion,



**Fig. 6.** The impact of the rate of abandonment ( $\eta$ ) and other system parameters on the expected number of EVs in the charging station ( $E_N$ ).



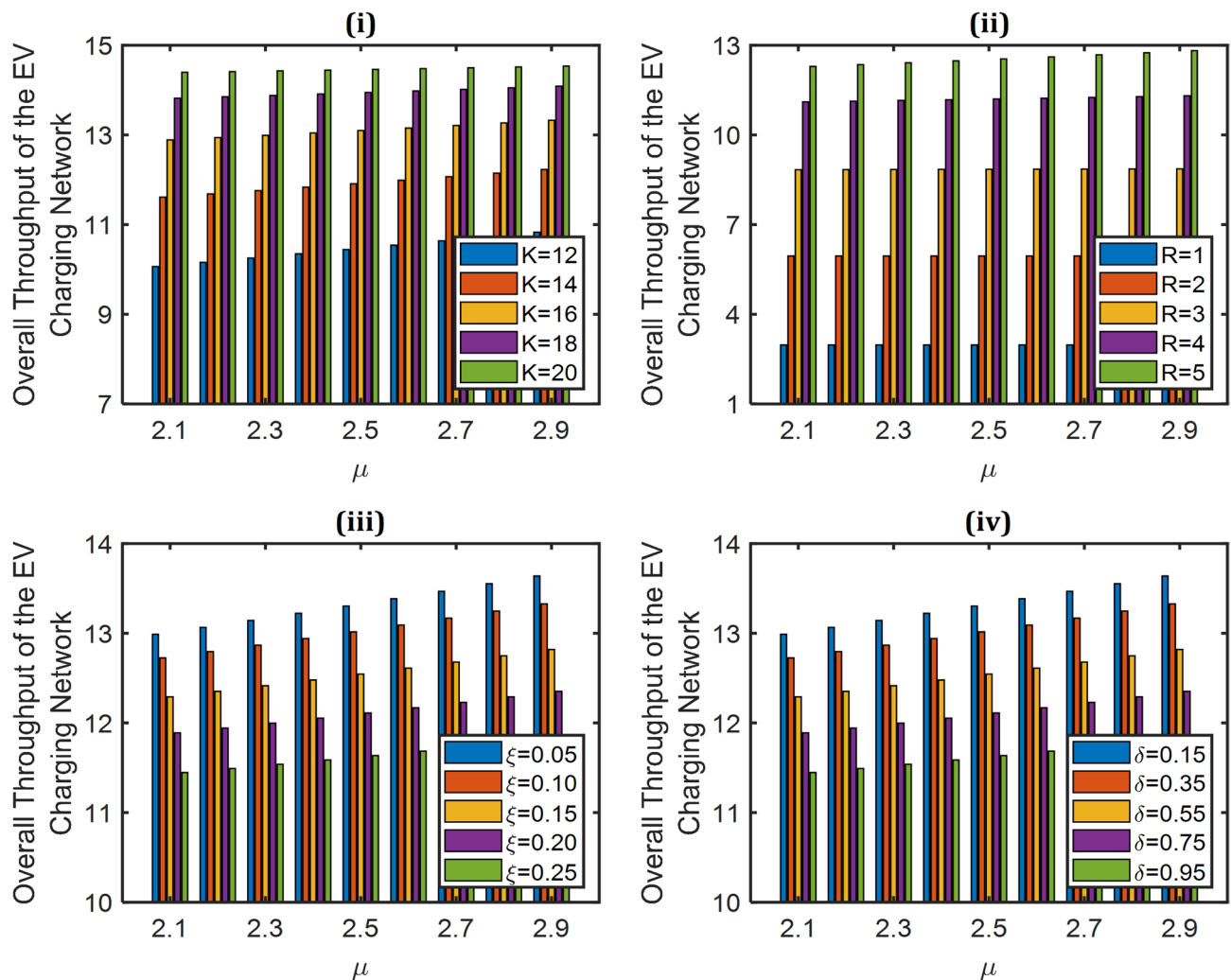
**Fig. 7.** The variation in the overall throughput ( $\tau_p$ ) of the EV charging network wrt arrival rate ( $\lambda$ ) and other system parameters.

one can observe the impact of service rate ( $\mu$ ) on the overall throughput of the EV infrastructure with the utilization of Fig. 8(i)–(iv). Therefore, appropriate service rates in EV charging infrastructures become essential for attaining optimal throughput, ensuring that charging stations can successfully manage variable demand while minimizing waiting times for service provisioning. This efficiency helps system designers and decision-makers by providing straightforward factors for planning modifications, enhancing customer satisfaction, and effectively allocating resources, thus promoting sustainable growth and enabling a higher EV adoption rate.

At multiple system parameter values, there are significant variations in the relationship between the expected number of busy servers ( $E_B$ ) and the arrival rate of individuals ( $\lambda$ ). In Fig. 9(i), higher system capacity ( $K$ ) values yield increased server utilization, with  $K = 20$  reaching approximately 4.7 expectation of busy servers at  $\lambda = 0.9$ , while  $K = 12$  shows lower utilization around 3.5. Figure 9(ii) illustrates that increased service channels ( $R$ ) significantly affect utilization patterns, with  $R = 1$  maintaining consistent utilization near 1.0 while  $R = 5$  reaches approximately 4.0 at higher arrival rates. Figure 9(iii) demonstrates that higher feedback probability ( $\xi$ ) values result in marginally increased server utilization, while Fig. 9(iv) shows minimal variation across different joining probability ( $\delta$ ) values, indicating this parameter has a negligible impact on server utilization.

The correlation between service rate ( $\mu$ ) and the expected number of busy servers ( $E_B$ ) demonstrates significant variations across different system parameters. Figure 10(i) illustrates that higher system capacity ( $K$ ) values consistently yield increased server utilization across all service rates, with  $K = 20$  maintaining the highest  $E_B$  values. In Fig. 10(ii), higher service channel ( $R$ ) values show increased server utilization, with  $R = 5$  reaching the highest  $E_B$  values while  $R = 1$  remains constant at  $E_B = 1$  regardless of service rate, indicating full utilization of the single server. Figure 10(iii) and (iv) reveal that feedback parameter ( $\xi$ ) has a moderate impact on server utilization, while joining probability ( $\delta$ ) demonstrates minimal influence on  $E_B$  values across different service rates.

There are predictable variations in the relation between the expected number of busy servers ( $E_B$ ) and the abandonment rate ( $\eta$ ) whenever the system parameters are updated. In Fig. 11(i), higher system capacity ( $K$ )

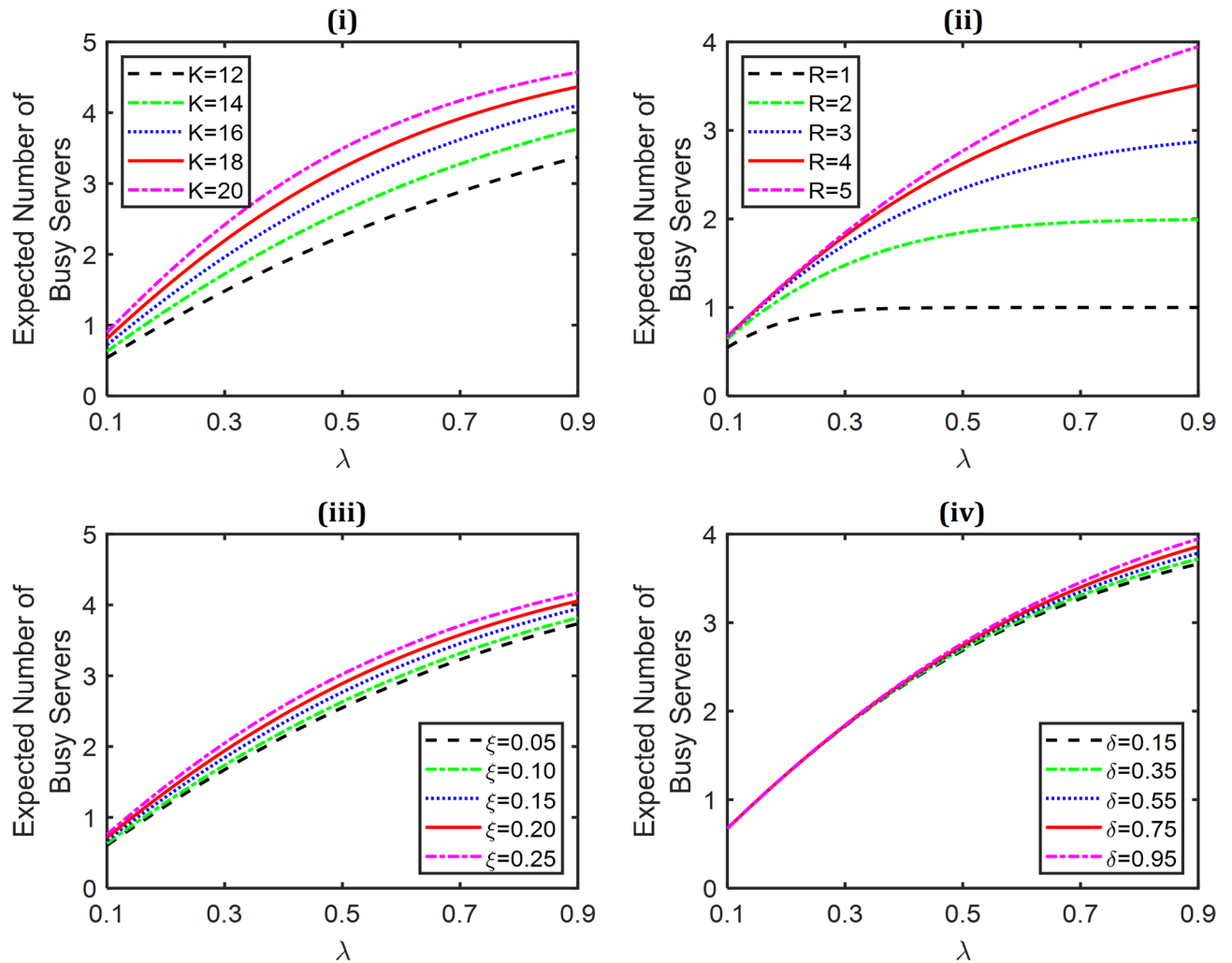


**Fig. 8.** The variation in the overall throughput ( $\tau_p$ ) of the EV charging network wrt service rate ( $\mu$ ) and other system parameters.

values maintain higher  $E_B$  levels, with  $K = 20$  exhibiting the highest utilization around 4.9 expectation value of the service providers, while all curves show slight downward trends as  $\eta$  increases. Figure 11(ii) reveals that increased service channels ( $R$ ) result in higher overall server utilization, with  $R = 5$  maintaining approximately 4.7 – 4.8 expectation of busy servers across all abandonment levels, while  $R = 1$  remains constant at  $E_B = 1$ , indicating abandonment has minimal impact on single-server configurations. Figure 11(iii) and (iv) demonstrate that higher feedback probability ( $\xi$ ) and joining probability ( $\delta$ ) values consistently yield increased server utilization, with all curves showing marginal decreases as abandonment rates increase. These comprehensive analyses of busy server patterns demonstrate logical consistency across parameter variations and reveal critical insights into resource utilization dynamics in EV charging infrastructures. The sensitivity of  $E_B$  to arrival rates, service configurations, and customer behavior parameters provides system designers with practical guidelines for optimizing operational efficiency and enhancing service quality in real-world implementations.

Regardless of the system characteristics, there are constant decreasing patterns in the correlation between the arrival rate ( $\lambda$ ) and the anticipated number of idle servers ( $E_I$ ). In Fig. 12(i), lower system capacity ( $K$ ) values maintain higher  $E_I$  levels at increased arrival rates, with  $K = 12$  showing approximately 1.8 value of the expectation of the idle servers at  $\lambda = 0.9$ , while  $K = 20$  approaches 0.5 idle servers. Figure 12(ii) illustrates that higher service channels ( $R$ ) result in more idle servers initially, with  $R = 5$  starting at approximately 4.5, expectation of idle servers at  $\lambda = 0.1$ , but all configurations show declining  $E_I$  as arrival rates increase. Figure 12(iii) and (iv) demonstrate that increased feedback probability ( $\xi$ ) values result in slightly fewer idle servers, while joining probability ( $\delta$ ) shows minimal differentiation at higher arrival rates, with all curves converging toward similar  $E_I$  values (approximately 1.1 – 1.2) at  $\lambda = 0.9$ .

The correlation between the service rate ( $\mu$ ) and the expected number of idle servers ( $E_I$ ) demonstrates distinctive patterns across varying system parameters. In Fig. 13(i), lower system capacity ( $K$ ) values yield significantly higher  $E_I$  levels, with  $K = 12$  showing an increase from 0.6 to 0.8 expected variation in idle servers as  $\mu$  increases from 2.1 to 2.9, while  $K = 20$  maintains the minimal number of expected idle servers

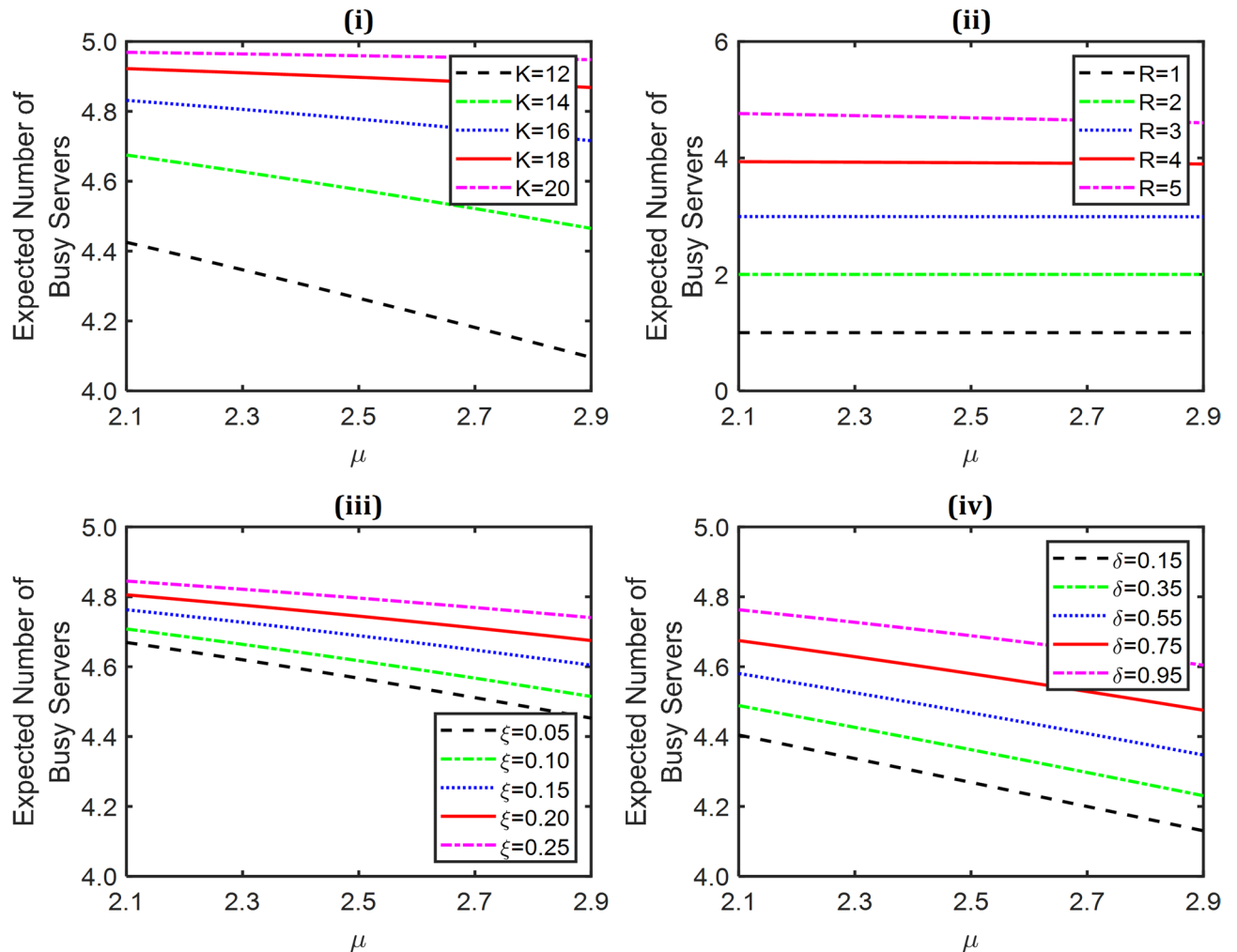


**Fig. 9.** The impact of arrival rate ( $\lambda$ ) and other system parameters on the expected number of busy servers ( $E_B$ ) in the charging station.

(below 0.1) across all service rates. Figure 13(ii) reveals that higher service channels ( $R$ ) result in more idle servers, with  $R = 5$  exhibiting a notable increase from 0.25 to 0.4, expectation of the idle servers, while  $R = 1$ ,  $R = 2$ , and  $R = 3$  maintain near-zero idle capacity regardless of service rate. Figure 13(iii) and (iv) demonstrate that lower feedback probability ( $\xi$ ) and joining probability ( $\delta$ ) values consistently yield more idle servers, with all configurations showing moderate upward trends as service rates increase.

Over a spectrum of system characteristics, the interrelationship between the abandonment rate ( $\eta$ ) and the anticipated number of idle servers ( $E_I$ ) shows patterns of consistent increment. In Fig. 14(i), lower system capacity ( $K$ ) values are consistently higher  $E_I$  levels, with  $K = 12$  showing approximately 0.75 value of the expected idle servers throughout all abandonment rates, while  $K = 20$  maintains minimal idle capacity (almost below 0.1). Figure 14(ii) illustrates that higher service channels ( $R$ ) result in more idle servers, with  $R = 5$  showing a gradual increase from 0.32 to 0.35 as  $\eta$  increases, while  $R = 1$ ,  $R = 2$ , and  $R = 3$  maintain near-zero idle servers regardless of abandonment rate. Figure 14(iii) and (iv) demonstrate that lower feedback probability ( $\xi$ ) and joining probability ( $\delta$ ) values consistently yield higher  $E_I$  levels, with all configurations showing slight upward trends as abandonment rates increase. The idle server patterns exhibit complementary behavior to busy server metrics, confirming mathematical validity while providing valuable perspectives on resource availability across varying operational conditions. These insights enable infrastructure planners to maintain optimal service levels by balancing idle capacity against demand fluctuations, ultimately supporting cost-effective resource allocation and capacity planning in practical EV charging deployments.

There are apparent trends in the relationship between arrival rate ( $\lambda$ ) and anticipated waiting time ( $W_S$ ), when system configurations are modified. In Fig. 15(i), different system capacity ( $K$ ) values show minimal variation in waiting times, with values remaining between 0.44 – 0.50 units across all arrival rates, indicating system capacity has a limited impact on customer waiting experience. Figure 15(ii) reveals that service channels ( $R$ ) significantly influence waiting times, with  $R = 1$  demonstrating substantially higher  $W_S$  values (up to 4 units) while  $R = 5$  maintains consistently lower waiting times across all arrival rates. Figure 15(iii) and (iv) illustrate that feedback probability ( $\xi$ ) and joining probability ( $\delta$ ) values have moderate effects on waiting

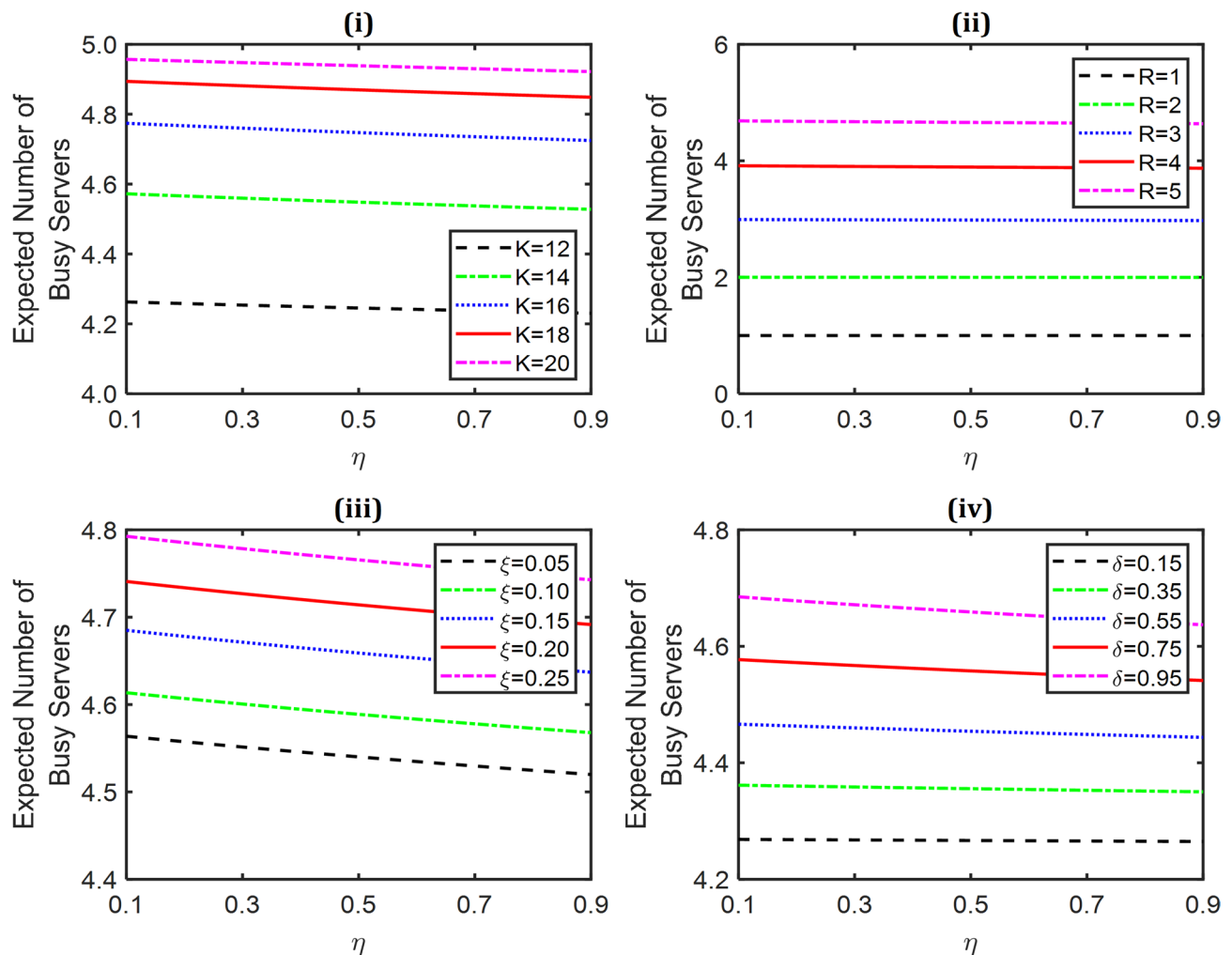


**Fig. 10.** The impact of service rate ( $\mu$ ) and other system parameters on the expected number of busy servers ( $E_B$ ) in the charging station.

times, with all configurations maintaining relatively stable  $W_S$  values between 0.4 – 0.55 units regardless of arrival rate fluctuations.

Further, there are consistent trends in the correlation between service rate ( $\mu$ ) and expected waiting time ( $W_S$ ), when system parameters are updated. In Fig. 16(i), higher system capacity ( $K$ ) values yield increased waiting times, with  $K = 20$  showing approximately 0.65 – 0.7 time units across all service rates, while  $K = 12$  maintains lower  $W_S$  values around 0.45 – 0.5 units. Figure 16(ii) reveals that service channels ( $R$ ) dramatically influence waiting times, with  $R = 1$  exhibiting substantially higher  $W_S$  values (approximately 3.5 units) while  $R = 5$  maintains minimal waiting times across all service rates. Figure 16(iii) and (iv) demonstrate that higher feedback probability ( $\xi$ ) and joining probability ( $\delta$ ) values consistently yield increased waiting times, with all configurations showing minimal sensitivity to service rate variations. From a logical perspective, these waiting time patterns align with queueing theory principles, demonstrating how service capacity constraints and customer behavior directly impact system congestion. The practical implications are significant for EV charging infrastructure planning, as they highlight how strategic adjustments to service channels and feedback mechanisms can meaningfully reduce customer waiting experiences while optimizing resource allocation.

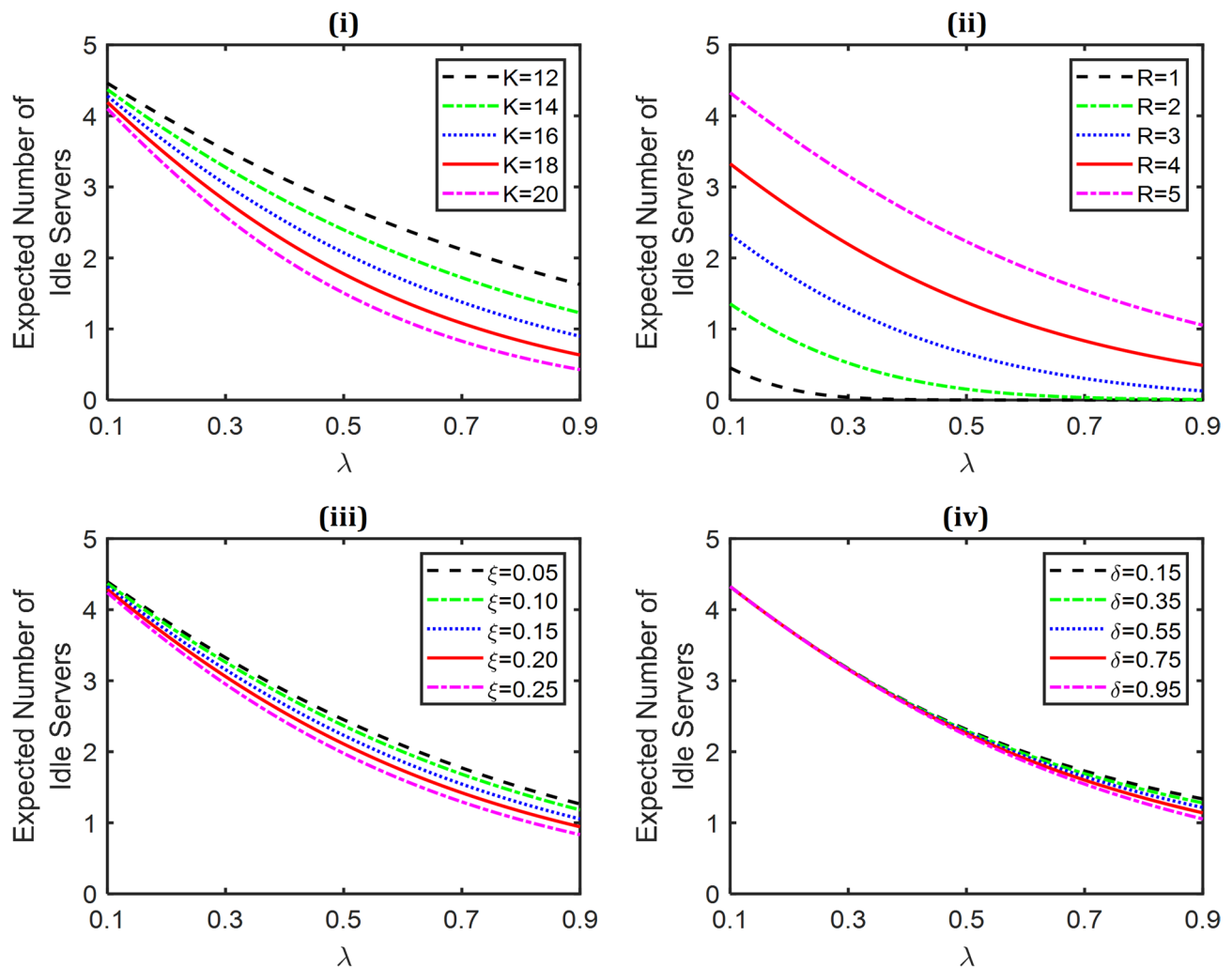
The investigation of system parameters ( $K, R, \lambda, \mu$ ) indicates substantial correlations with various system performance measures, as illustrated in Table 3. In Table 3, one can observe that when the system capacity ( $K$ ) increases from 15 to 25, both the anticipated number of electric vehicles in the EV infrastructure ( $E_N$ ) and the expected number of EVs waiting for their turn ( $E_Q$ ) increase significantly from 6.17 to 14.98 and 3.23 to 11.98, respectively. In contrast, increasing the number of service providers ( $R$ ) from 3 to 7 results in a slight decrease in  $E_N$  (from 6.16 to 5.03) and a substantial reduction in  $E_Q$  (from 3.23 to 0.12), indicating enhanced queueing management of the EV infrastructure (cf. Table 3). In addition, increased arrival rates ( $\lambda$ ) ranging from 1.0 to 2.0 result in enhanced  $E_N$  (6.17 to 10.12) and  $E_Q$  (3.23 to 7.12), whereas fluctuations in service rate ( $\mu$ ) from 2.0 to 3.0 exhibit negligible influence, with  $E_N$  reducing slightly from 6.17 to 6.02. Similarly, Table 4 illustrates the influence of operational parameters ( $\mu_1, \xi, \delta, \eta$ ) on system performance. It is observed that as the fast-charging rate ( $\mu_1$ ) is increased from 3.0 to 4.0, the overall throughput of the



**Fig. 11.** The impact of abandonment rate ( $\eta$ ) and other system parameters on the expected number of busy servers ( $E_B$ ) in the charging station.

EV infrastructure ( $\tau_p$ ) increases from 10.66 to 11.31, while  $E_N$  and  $E_Q$  decrease from 4.97 to 4.69 and 0.54 to 0.34, respectively. This happens because of the fact that fast service of the service provider significantly reduces the high congestion and improves the overall throughput of the EV infrastructure. Besides, one can observe that as the feedback probability ( $\xi$ ) increases, the expected length of the EV infrastructure ( $E_N$ ) increases from 4.97 to 5.61. Similarly, the joining probability ( $\delta$ ) significantly influences  $E_N$ , with values increasing from 0.5 to 0.9, resulting in  $E_N$  escalating from 4.97 to 6.32. Moreover, the reneging rate ( $\eta$ ) has negligible effects, as  $E_N$  shows a minor decrease from 4.97 to 4.93 as  $\eta$  increases from 0.1 to 0.3. These results highlight that the factors that exhibit a significant impact on system performance indicators are customer impatience, feedback constraints, and system capacity. By offering in-depth analytics for management scheduling, improving customer satisfaction, and efficiently allocating resources, these results are helpful to system designers and decision-makers in promoting sustainable development and facilitating electric vehicle adoption.

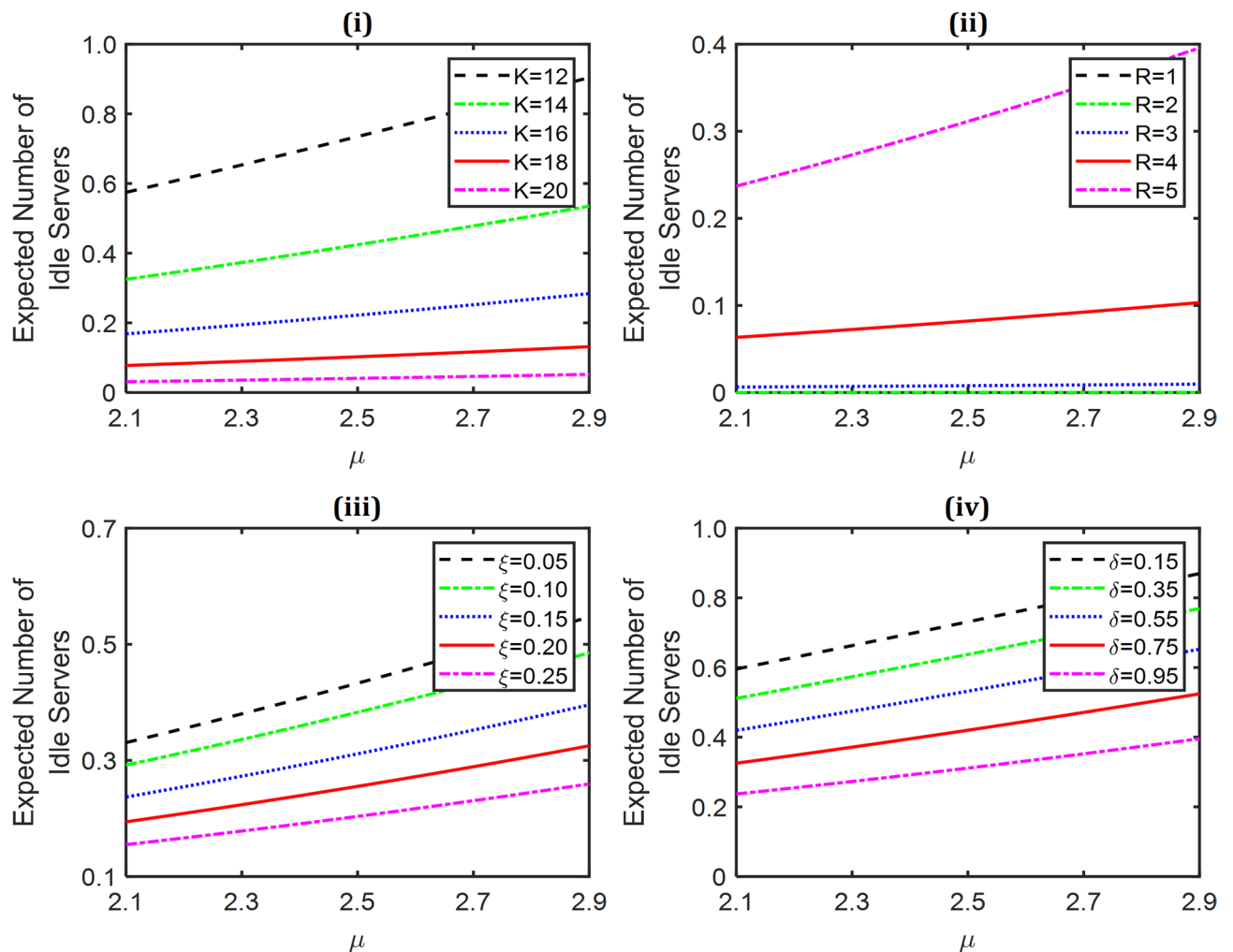
Further, Table 5 illustrates the numerical results based on the complexity analysis, offering a profound insight into the complex relationships between system parameters such as arrival rate ( $\lambda$ ), service rate ( $\mu$ ), and the number of servers ( $R$ ) within the developed queueing framework. The analysis highlights the framework's complexity by demonstrating how varying these parameters directly impacts key performance metrics, including the expected number of EVs in the system ( $E_N$ ), expected waiting time ( $W_S$ ), and system utilization ( $\rho$ ). For instance, increasing the value of active service providers ( $R$ ) from 3 to 7 at constant  $\mu$  significantly reduces waiting times (e.g.,  $W_S$  decreases from 0.73210 to 0.55374 for  $\lambda = 1.0$ ) while maintaining high service quality. This dynamically balances resource allocation against congestion, reflecting the model's versatility to adapt to fluctuating demand. Similarly, with the other combinations, the observed convex trends further highlight how optimized configurations minimize queueing costs, showcasing practical relevance for deploying cost-effective EV charging networks. By addressing real-world challenges like congestion control, scalability, and customer satisfaction, Table 5 emphasizes the model's potential to guide infrastructure planning and maintain operational efficiency in high-demand charging circumstances.



**Fig. 12.** The impact of arrival rate ( $\lambda$ ) and other system parameters on the expected number of idle servers ( $E_I$ ) in the charging station.

The study of the anticipated total cost in Fig. 17 indicates that the convex trend of total cost ( $TC$ ) determines the optimal values of service rate ( $\mu$ ) and server capacity ( $R$ ) that minimize the cost of operations. Figure 17(i) demonstrates that the minimum total cost, approximately 1700 units, is observed at  $\mu \approx 11$ , indicating that this service rate balances service efficiency and operating costs optimally. Meanwhile, the Fig. 17(ii) indicates that the lowest  $TC$  occurs around  $R = 4$ , where the system effectively reduces waiting times without acquiring excessive maintenance costs for additional servers. Beyond these levels, total cost increases due to decreased profitability on service enhancements or increasing operational expenses. These findings emphasize the importance of fine-tuning system parameters. By maintaining  $\mu \approx 11$  and  $R = 4$ , the EV charging network operators can achieve the most economical configuration, minimizing customer queueing delays while keeping resource utilization optimum. These results highlight the practical relevance of the queueing framework, offering EV station planners valuable insights into designing and managing charging infrastructures to meet demand while ensuring cost-effective operations.

Moreover, the surface plots presented in Figs. 18 and 19 provide a comprehensive visualization of how multiple system parameters jointly influence the expected total cost ( $TC$ ) associated with the EV charging infrastructures. In Fig. 18, we observe how the arrival rate ( $\lambda$ ) interrelates with different system parameters. Figure 18(i) reveals that increasing both  $\xi$  (probability associated with the feedback mechanism) and  $\lambda$  simultaneously results in the incremental change in  $TC$  from approximately 1700 to 4100 units, indicating how customer impatience is enhanced with high demand to escalate operational costs. Similarly, Fig. 18(ii) shows  $\delta$  (balking probability) and  $\lambda$  together increasing costs from 1600 to 2200 units, while Fig. 18(iii) demonstrates that higher  $\eta$  (parameter associated with independent abandonments) values can partially mitigate the cost impact of increasing  $\lambda$ . Particularly, Fig. 18(iv) illustrates how increasing  $\mu_1$  (service rate threshold) from 3.1 to 3.9 effectively mitigates increasing arrival rates, keeping  $TC$  between 1700 and 2300 units. Similarly, Fig. 19 examines the interrelationship between service rate ( $\mu$ ) and other variables. Figure 19(i) shows that when both  $\xi$  and  $\mu$  increase,  $TC$  ranges from 2000 to 4000 units, with costs increasing more significantly at higher abandonment rates. Further, Fig. 19(ii) demonstrates that increasing  $\mu$  from 2.1 to 2.9 while keeping



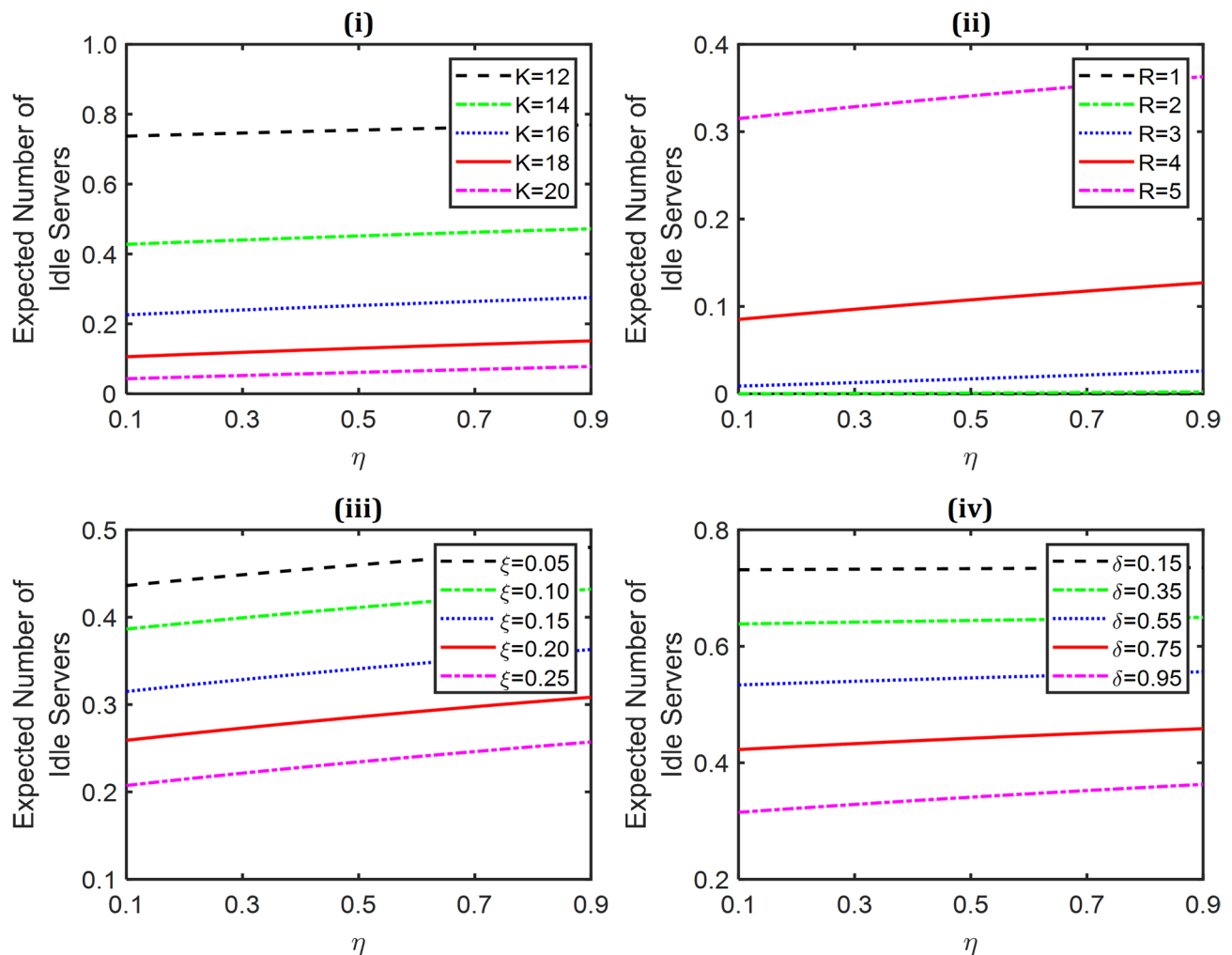
**Fig. 13.** The impact of service rate ( $\mu$ ) and other system parameters on the expected number of idle servers ( $E_I$ ) in the charging station.

$\delta$  minimum maintains  $TC$  between 1600 and 2000 units, offering substantial cost strategies. The relationship between  $\mu$  and  $\eta$  in Fig. 19(iii) reveals a nearly smooth surface at higher  $\mu$  values (around 2.9), suggesting that improved service rates reduce the impact of independent abandonment on total cost. Finally, Fig. 19(iv) highlights the inverse relationship between  $\mu$  and  $\mu_1$  where  $TC$  decreases from 2050 to 1900 units when both parameters increase, demonstrating how dynamic threshold policies can optimize system performance across varying operational conditions. These multidimensional analyses provide charging station operators with significant insights for fine-tuning operational parameters to achieve cost-efficient configurations adaptable to fluctuating demand patterns.

### Significant managerial insights

The integration of various customer and server behavior patterns, namely, balking, abandonment, service threshold policies, and feedback mechanisms, in EV charging infrastructure provides significant operational insights for system managers and service providers. The model's thorough integration of these behaviors simulates real-world circumstances in which customers may choose not to join lengthy queues (balking), abandon because of prolonged waiting periods (reneging), return to the EV network to complete unsatisfactory services (feedback mechanism), and the switching behavior of the service provider to a faster service rate to reduce the overload on the EV infrastructure. This multidimensional strategy enables charging network operators to design more advanced strategies for capacity management, congestion control, and service quality enhancement while minimizing the rate of lost individuals through multiple departure mechanisms.

The analysis of customer behavior patterns provides valuable operational insights for infrastructure planning and resource allocation. The interrelationship between service rates and system throughput indicates the fundamental requirement for balanced capacity management, while the influence of customer abandonment rates highlights the importance of queueing management procedures. System managers should establish dynamic threshold policies that can anticipate fluctuating customer tolerance levels and arrival patterns. The integration of feedback mechanisms into the model emphasizes the significance of service quality assessment



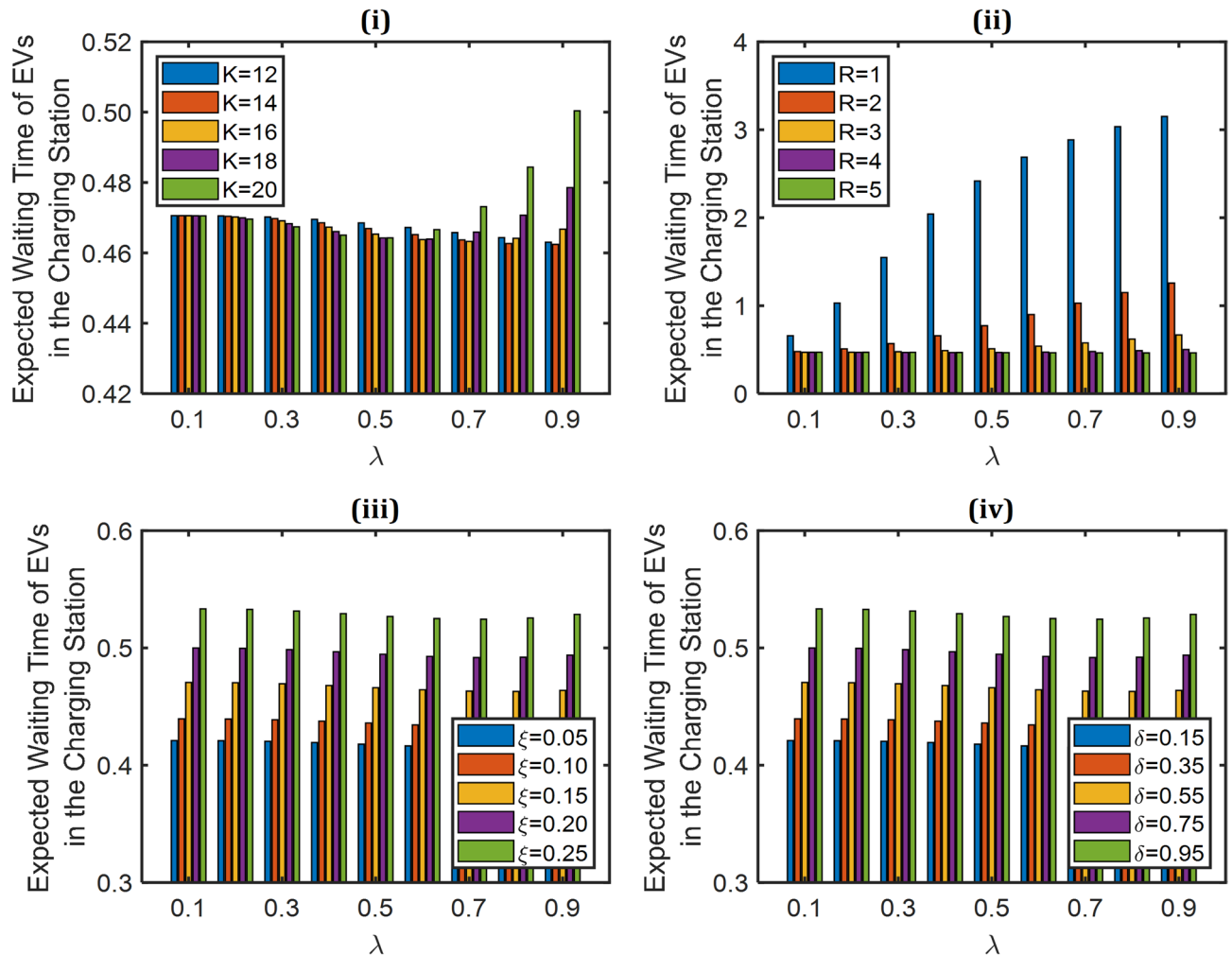
**Fig. 14.** The impact of abandonment rate ( $\mu$ ) and other system parameters on the expected number of idle servers ( $E_I$ ) in the charging station.

and customer satisfaction monitoring. Additionally, the study demonstrates that improving server allocation based on predicted individual behavior patterns can significantly reduce system abandonment rates and enhance overall operational efficiency. These insights enable decision-makers to build more robust and customer-centric operational strategies that balance service quality with resource utilization while ensuring system stability under fluctuating demand patterns.

The numerical investigation reveals several critical insights for operational decision-making. System capacity enhancement demonstrates a twofold impact; while it boosts the overall service capability, it simultaneously leads to disproportionate growth in queue lengths, thereby improving service quality. In contrast, adopting appropriate feedback restriction mechanisms proves highly advantageous, indicating that even with minor changes in total system occupancy, queue lengths can be significantly decreased, leading to better operational efficiency. This demonstrates that advanced queue management strategies may be more effective than fundamental capacity enhancement. The analysis of arrival and service rates further underlines the fact that while higher arrival rates predictably lead to higher system congestion, merely modifying service rates demonstrates limited advantages in improving system performance. These relationships emphasize that optimal system performance is achieved not by making confined parameter adjustments, but through an integrated approach that prioritizes queue management strategies and feedback mechanisms, allowing system designers and operators to maintain service quality while efficiently utilizing available resources.

### Conclusion and future perspectives

The current investigation contributes to the EV-based queueing literature by performing an in-depth analysis of EV charging networks utilizing a complex queueing framework that integrates numerous customer and server behaviors such as balking, abandonment, feedback mechanism, and service threshold policies. The study proposes a novel mathematical and statistical framework for analyzing finite population-based multiple-server EV charging infrastructure while incorporating dynamic customer behavior patterns. In addition, an adequate mathematical framework for interpreting system dynamics has been constructed through the formulation of

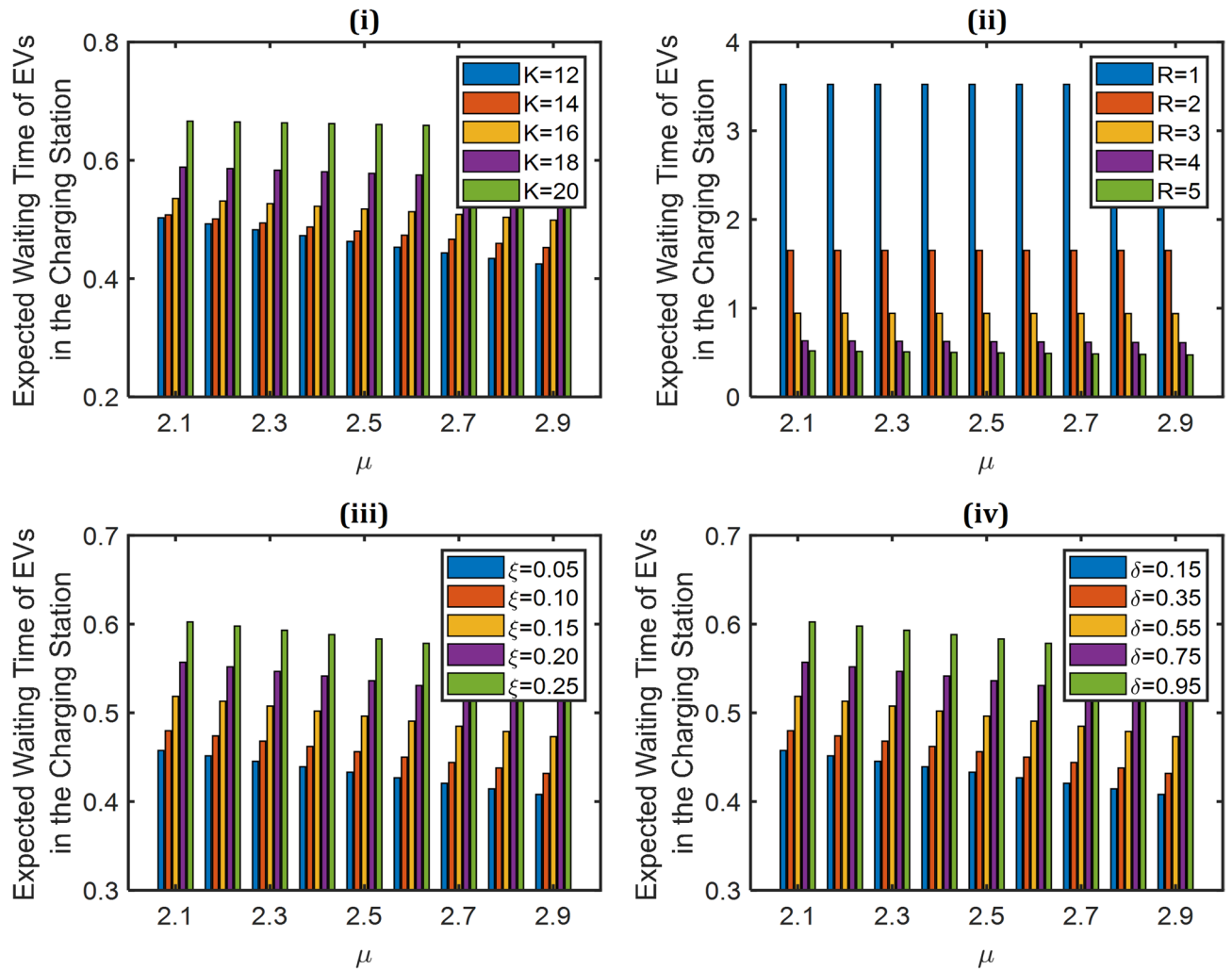


**Fig. 15.** The variation in the expected waiting time ( $W_S$ ) of EVs in the charging station wrt arrival rate ( $\lambda$ ) and other system parameters.

Chapman-Kolmogorov differential-difference equations and the application of Markovian strategies. The matrix solution technique has been efficiently implemented to derive steady-state probability distributions, enabling visualization of significant performance indicators such as expected queue lengths, server utilization, and the overall system throughput.

The significant contribution comprises the impact of customer abandonment rates and tolerance rates based on system performance and resource utilization, as well as the impact of service quality in controlling customer satisfaction levels and feedback mechanisms. Through comprehensive numerical analysis and graphical representations, we have demonstrated how system characteristics such as arrival rates and service rates substantially affect operating efficiency. The proposed queueing framework offers charging network operators practical tools to understand how they may optimize their service operations while dealing with different types of customer departures and service redundancies. The statistical and numerical results support the model's real-life applicability and provide beneficial insights for capacity planning and allocating resources in EV charging infrastructures. The research presents potential applications in numerous technical sectors where customer behavior and quality-of-service (QoS) become one of the fundamental components.

Future research directions include extending the model to integrate more complex service time distributions, such as the Weibull distribution, the Erlang distribution, and the hyper-exponential distribution. The framework could be extended by implementing advanced queueing strategies, including bulk arrival and service procedures, and Markovian arrival process (MAP) strategies. Other queueing nomenclatures and complex service procedures could be incorporated to represent more intricate system characteristics. Additionally, investigating the integration of emerging technologies, including artificial intelligence, machine learning, real-time analytics, and smart grid technologies, might further improve the model's predictive capabilities and operational efficiency. These statistical and technological advancements would contribute to establishing more reliable and customer-centric EV charging infrastructures, thereby promoting the sustainable transformation to green transportation networks.



**Fig. 16.** The variation in the expected waiting time ( $W_S$ ) of EVs in the charging station wrt service rate ( $\mu$ ) and other system parameters.

| $(K, R, \lambda, \mu)$ | $E_N$     | $E_Q$     | $E_B$    | $\tau_p$ | $W_S$    |
|------------------------|-----------|-----------|----------|----------|----------|
| (15, 3, 1.0, 2.0)      | 6.166943  | 3.227105  | 2.939838 | 8.262297 | 0.732099 |
| (20, 3, 1.0, 2.0)      | 10.285321 | 7.289264  | 2.996056 | 8.867531 | 1.114095 |
| (25, 3, 1.0, 2.0)      | 14.976675 | 11.976762 | 2.999913 | 8.923410 | 1.572809 |
| (15, 5, 1.0, 2.0)      | 5.312592  | 0.661063  | 4.369940 | 9.660186 | 0.519022 |
| (15, 7, 1.0, 2.0)      | 5.031003  | 0.118635  | 5.193957 | 9.588038 | 0.553743 |
| (15, 3, 1.5, 2.0)      | 8.608232  | 5.614100  | 2.994131 | 8.832563 | 0.944582 |
| (15, 3, 2.0, 2.0)      | 10.122608 | 7.123258  | 2.999350 | 8.911643 | 1.092233 |
| (15, 3, 1.0, 2.5)      | 6.094840  | 3.176793  | 2.918046 | 8.343299 | 0.716860 |
| (15, 3, 1.0, 3.0)      | 6.016401  | 3.123003  | 2.893398 | 8.431251 | 0.700608 |

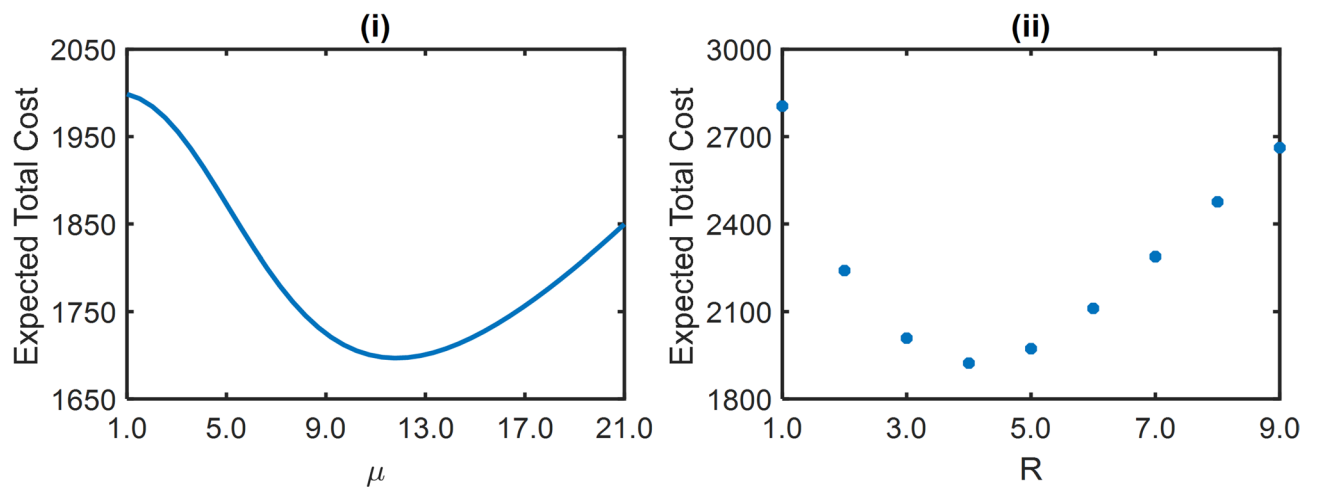
**Table 3.** The impact of system parameters  $K$ ,  $R$ ,  $\lambda$ , and  $\mu$  on various system performance measures.

| $(\mu_1, \xi, \delta, \eta)$ | $E_N$    | $E_Q$    | $E_B$    | $\tau_p$  | $W_S$    |
|------------------------------|----------|----------|----------|-----------|----------|
| (3.0, 0.1, 0.5, 0.1)         | 4.967765 | 0.544856 | 4.422909 | 10.664093 | 0.463472 |
| (3.5, 0.1, 0.5, 0.1)         | 4.799797 | 0.420671 | 4.379126 | 11.044841 | 0.432925 |
| (4.0, 0.1, 0.5, 0.1)         | 4.686909 | 0.339673 | 4.347237 | 11.314124 | 0.413013 |
| (3.0, 0.2, 0.5, 0.1)         | 5.254000 | 0.709716 | 4.544284 | 9.858352  | 0.529140 |
| (3.0, 0.3, 0.5, 0.1)         | 5.605286 | 0.944090 | 4.661196 | 8.977973  | 0.617841 |
| (3.0, 0.1, 0.7, 0.1)         | 5.562736 | 1.011044 | 4.551692 | 11.296950 | 0.488043 |
| (3.0, 0.1, 0.9, 0.1)         | 6.320007 | 1.643253 | 4.676754 | 11.911522 | 0.523359 |
| (3.0, 0.1, 0.5, 0.2)         | 4.950176 | 0.530953 | 4.419223 | 10.645981 | 0.460389 |
| (3.0, 0.1, 0.5, 0.3)         | 4.933685 | 0.517988 | 4.415698 | 10.628657 | 0.457498 |

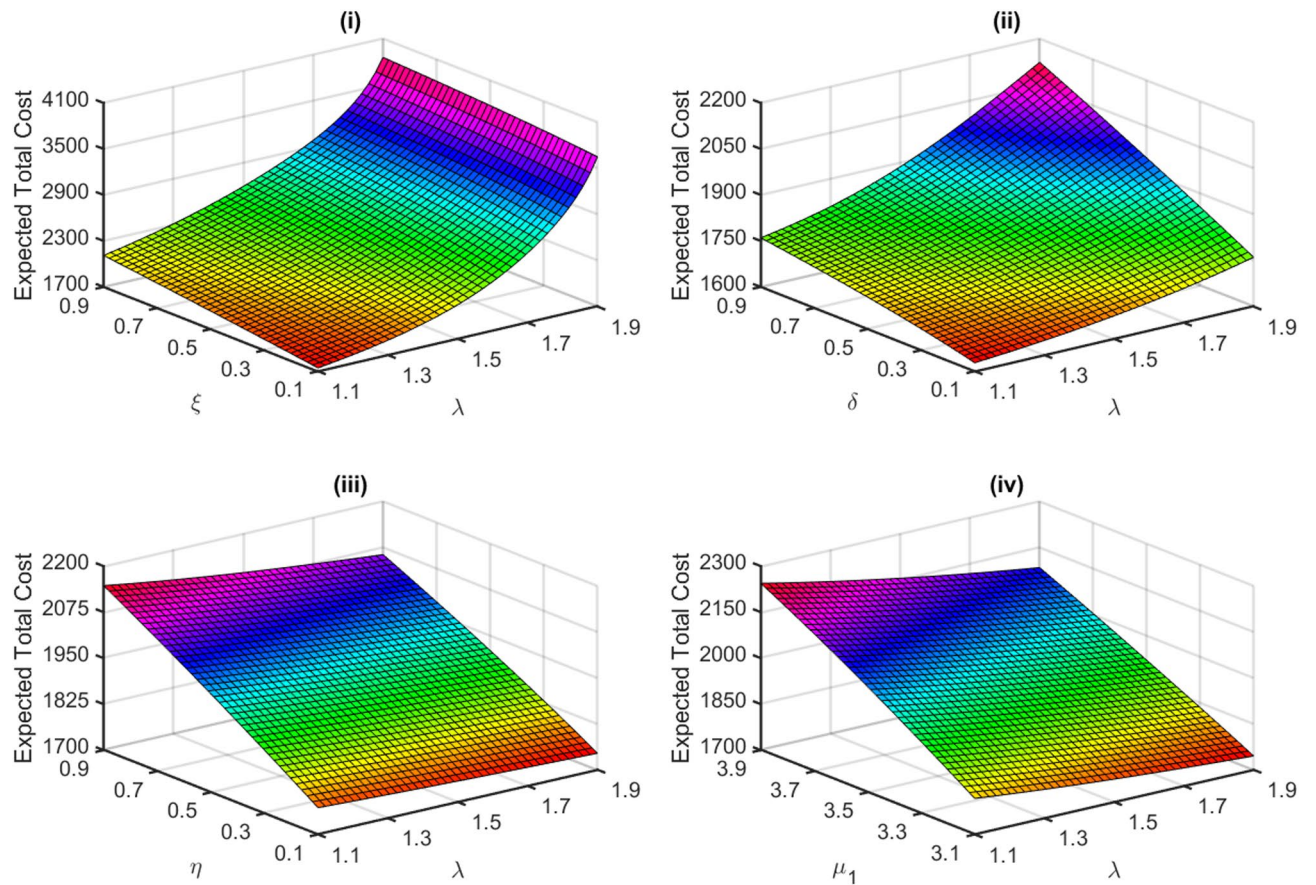
**Table 4.** The impact of system parameters  $\mu_1$ ,  $\xi$ ,  $\delta$ , and  $\eta$  on various system performance measures.

| $(\lambda, \mu, R)$ | $E_N$   | $E_Q$   | $W_S$   | $\rho = \left(\frac{\lambda}{R\mu}\right)$ |
|---------------------|---------|---------|---------|--------------------------------------------|
| (1.0, 2.0, 3)       | 6.16694 | 3.22710 | 0.73210 | 0.66667                                    |
| (1.0, 2.0, 5)       | 5.03100 | 0.66106 | 0.51902 | 0.40000                                    |
| (1.0, 2.0, 7)       | 5.31259 | 0.11863 | 0.55374 | 0.28571                                    |
| (1.0, 2.5, 3)       | 6.09484 | 3.17679 | 0.71686 | 0.83333                                    |
| (1.0, 2.5, 5)       | 4.69382 | 0.56017 | 0.46600 | 0.50000                                    |
| (1.0, 2.5, 7)       | 4.72886 | 0.07475 | 0.46306 | 0.35714                                    |
| (1.0, 3.0, 5)       | 4.35159 | 0.46792 | 0.41629 | 0.60000                                    |
| (1.0, 3.0, 7)       | 4.21985 | 0.04666 | 0.39279 | 0.42857                                    |
| (1.5, 2.0, 5)       | 6.45163 | 1.67164 | 0.52392 | 0.60000                                    |
| (1.5, 2.0, 7)       | 6.47045 | 0.40778 | 0.51666 | 0.42857                                    |
| (1.5, 2.5, 5)       | 6.26513 | 1.57648 | 0.49633 | 0.75000                                    |
| (1.5, 2.5, 7)       | 6.01548 | 0.31825 | 0.45347 | 0.53571                                    |
| (1.5, 3.0, 5)       | 6.05746 | 1.47581 | 0.46722 | 0.90000                                    |
| (2.0, 2.5, 7)       | 7.00313 | 0.72675 | 0.44936 | 0.71429                                    |
| (2.0, 3.0, 7)       | 6.65038 | 0.62331 | 0.40679 | 0.85714                                    |

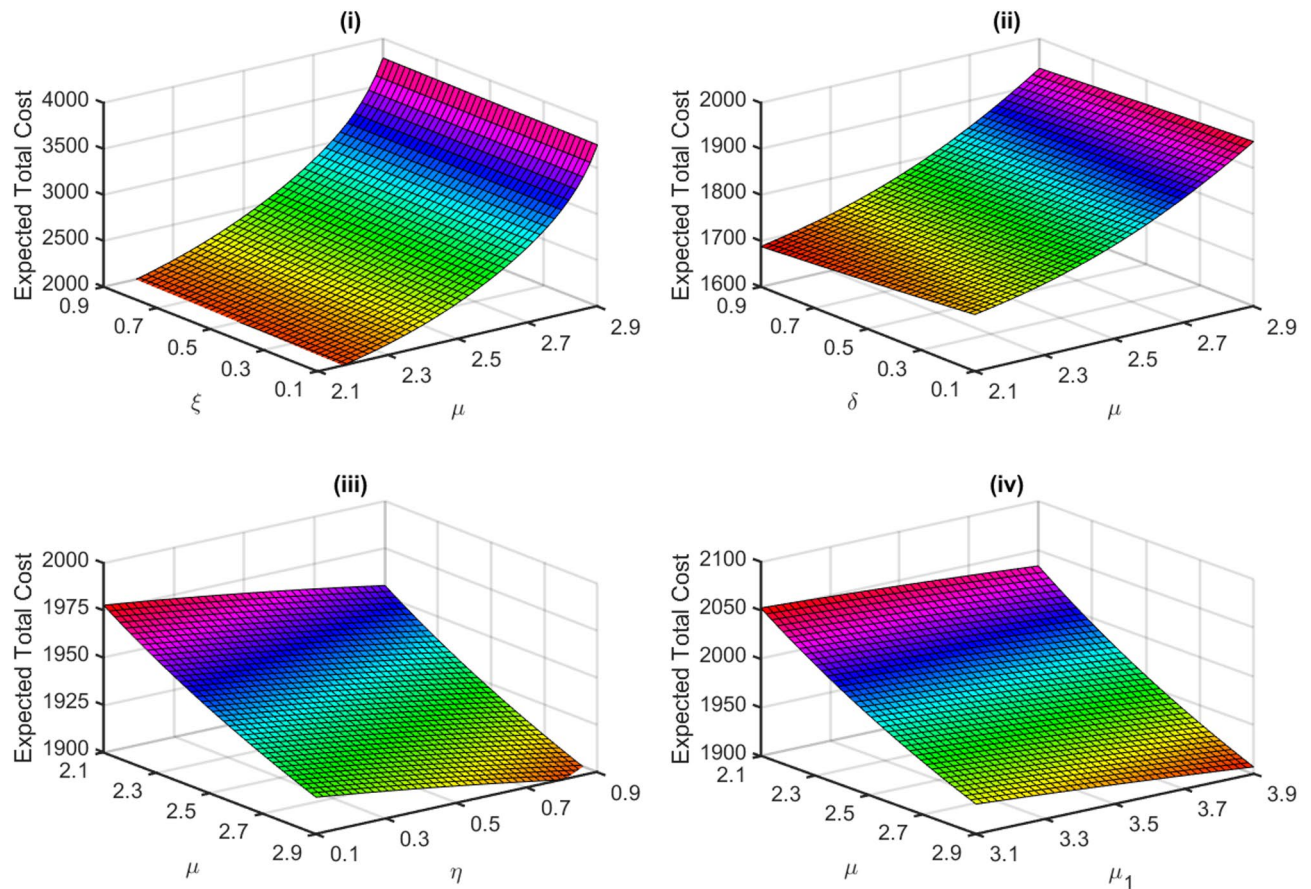
**Table 5.** Complexity analysis for the developed queueing framework utilizing multiple queueing-based performance measures.



**Fig. 17.** Convex plot for the expected total cost ( $TC$ ) wrt decision parameters  $\mu$  and  $R$ .



**Fig. 18.** The variation in the expected total cost ( $TC$ ) associated with the charging station wrt arrival rate ( $\lambda$ ) and other system parameters.



**Fig. 19.** The variation in the expected total cost ( $TC$ ) associated with the charging station wrt service rate ( $\mu$ ) and other system parameters.

### Data availability

The datasets used and/or analysed during the current study are available from the corresponding author on reasonable request.

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### Declarations

### Competing interests

The authors declare no competing interests.

### Additional information

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