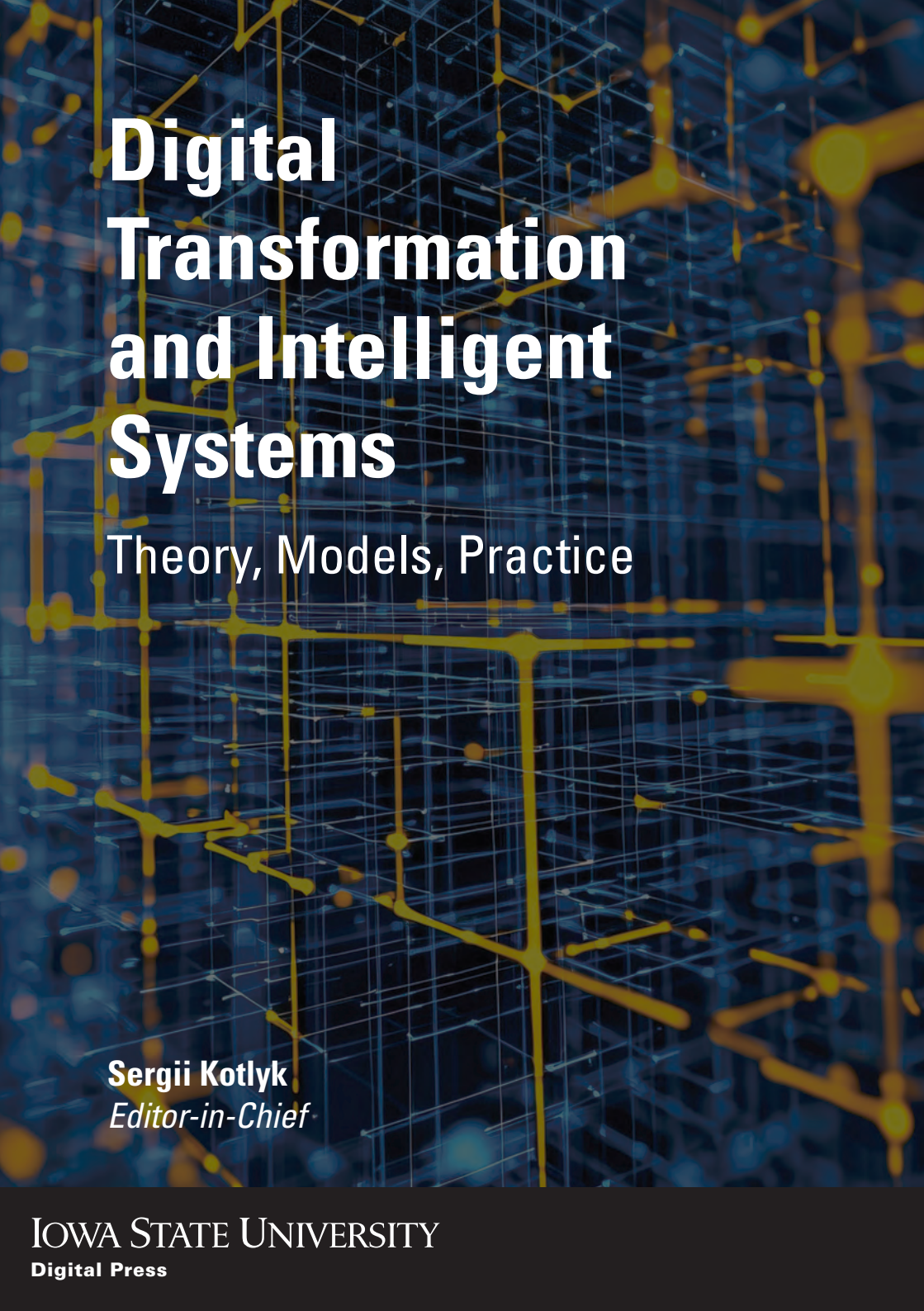




Digital Transformation and Intelligent Systems

Theory, Models, Practice

Sergii Kotlyk
Editor-in-Chief

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DIGITAL TRANSFORMATION AND INTELLIGENT SYSTEMS: THEORY, MODELS, PRACTICE

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Sergii Kotlyk

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This monograph summarizes and analyzes the current state of scientific research in the field of information technology, mathematical modeling, automation of production processes, blockchain architectures and artificial intelligence. It considers issues of system optimization, the use of ontologies in architectural modeling, process control, the development of new visualization methods, and the use of large language models for automatic data processing.

The monograph will be useful for both specialists and employees of companies working in the IT field, as well as teachers, masters, students and post-graduates of higher educational institutions, as well as everyone interested in modern trends in the field of digital technology.

The monograph was prepared based on the results of the XVII International Scientific and Practical Conference “Information Technology and Automation — 2024”, held in October 2024 at the Odessa National Technological University.

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Editor-in-Chief **Sergii Kotlyk**, Ph.D., Associate Professor, Odesa National University of Technology (Odessa, Ukraine)

Team of Authors: **Brynza Natalia, Chaplinskyy Yuriy, Kaverinskiy Vladislav, Khoshaba Oleksandr, Kotlyk Sergii, Malakhov Kyrylo, Romaniuk Oksana, Romanyuk Oleksandr, Shvets Dmitriy, Sokolova Oksana, Stepanov Michael, Voinova Svitlana, Zavalniuk Yevhen**

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6. DEVELOPMENT OF NEW BIDIRECTIONAL REFLECTANCE DISTRIBUTION FUNCTIONS FOR HIGHLY REALISTIC IMAGES FORMATION

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Oleksandr Romanyuk, Yevhen Zavalniuk, Oksana Romanyuk

In the work, the development of new bidirectional reflectance distribution functions to ensure the formation of highly realistic three-dimensional images is discussed. The features of the developed bidirectional reflectance distribution functions, which lie in the improvement and approximation of existing functions, the combination of individual functions, and in the calculation of normalizing coefficients for the functions, are analyzed. Recommendations for adaptive selection of bidirectional reflectance distribution functions are provided.

6.1. INTRODUCTION

The main requirements for object rendering [1] are high speed and a sufficient level of realism.

To create realistic three-dimensional graphic images, it is necessary to take into account the features of light reflection from the surfaces of scene objects. In particular, it is necessary to highly accurately reproduce the features of glare, both in the epicenter zone and in the attenuation zone. Depending on the problem, microfacet surface representation and corpuscular-wave dual theory can be used. In addition, physically correct models [2] of light reflection must comply with the law of conservation of energy and the Helmholtz reciprocity principle. The Helmholtz reciprocity principle lies in the possibility of swapping the direction of incoming and outgoing light. The law of conservation of energy means that the amount of light reflected by the surface cannot be greater than the amount of light incident on the surface.

The process of finding the specular component of color, which is responsible for glare, is the most time-consuming [1], since it involves calculating at each point of the surface normalized vectors to the observer, to the light source and the normal vector. The use of the most complex lighting models and algorithms during rendering complicates highly realistic visualization of

the scene in real time. Accordingly, a high speed of visualization of objects is ensured by using simple forms of light reflection models [3]. Simple light reflection models are characterized by a low degree, a minimum number of complex operations (division, finding the root, calculating inverse trigonometric functions), and the possibility of hardware implementation.

Existing surface reflection models do not always meet the requirements of high realism and high performance.

Therefore, it is relevant to develop light reflection models that provide highly realistic reproduction of glare and can be used in highly productive rendering systems.

6.2. BIDIRECTIONAL REFLECTANCE DISTRIBUTION FUNCTIONS

Bidirectional reflectance distribution functions (BRDFs) [2,4] represent the optical properties of materials.

The BRDF determines what fraction of the light that arrived from the direction $\vec{L}$ will be reflected in the direction $\vec{V}$. The BRDF is calculated by the formula [2,4]

$$BRDF = \frac{dL_r(\vec{V})}{L_i(\vec{L})\cos(\theta_i)d\omega_i},$$

where $d\omega$ is the differential solid angle within which the radiant flux falls, L_r is the radiance in the reflected direction, $L_i(\vec{L})$ is the irradiance, θ_i is the angle between $\vec{L}$ and $\vec{N}$.

The methods that are used for illumination modelling operate with ambient and reflected light. Reflected light has two components: diffuse and specular.

Ambient (background) [2] light is light that is reflected from the environment, neighboring objects.

If there is a point light source in the scene, the intensity of diffuse reflection is proportional to the cosine of the angle between the normal to the surface $\vec{N}$ and the direction towards the light source $\vec{L}$. In this case, Lambert's law of cosines [2] is used to calculate the intensity of diffuse reflection. The expression is used

$$\frac{k_d}{\pi} \cdot \cos(\theta_i), \quad (6.1)$$

where k_d is the diffuse reflection coefficient.

A common modification of the Lambert model [5] is to use the Fresnel coefficient F to take into account the fact that the reflected light is unavailable for subsurface scattering. As a result, an additional multiplier is introduced

$$(1 - F(\vec{H}, \vec{L})) , \quad (6.2)$$

where $\vec{H} = (\vec{L} + \vec{V}) / |\vec{L} + \vec{V}|$, $\vec{V}$ — vector to the observer.

The specular color component is used for glare reproduction.

BRDFs are mostly associated with the specular color component. In addition, some BRDFs (for example, Lambert BRDF) or BRDF components are used with the diffuse color component.

In general, BRDFs are divided into physically-accurate and empirical [2,4].

Physically-accurate BRDFs are more accurate and usually take into account the division of the object's surface into microfacets. The physically-accurate BRDFs are usually based on the Cook-Torrance model.

In the Cook-Torrance model [2,4,6] it is taken into account that the reflectivity of the surface is affected by microfacets, oriented along the vector $\vec{H} = (\vec{L} + \vec{V}) / |\vec{L} + \vec{V}|$, where $\vec{L}$ and $\vec{V}$, respectively, are the vectors to the light source and the observer. The model includes a diffuse component, represented by Lambertian reflection, and a specular component. During the calculation of the specular component, the Fresnel factor F , the geometric shading factor G , and the distribution of microfacets D are used. The specular component of this model is calculated according to the formula [2,4,6]

$$\frac{FDG}{4\cos(\theta_i)\cos(\theta_o)} , \quad (6.3)$$

where θ_o is the angle between $\vec{V}$ and $\vec{N}$.

A popular modification of the model is the Disney BRDF [7]. The model includes the usage of the GGX distribution as D for the main specular lobe

$$\frac{\alpha_g^2}{\pi} \frac{1}{(1 + (\alpha_g^2 - 1)\cos(\gamma)^2)^2} , \quad (6.4)$$

where $\alpha_g = \alpha^2$ (α — roughness), γ — angle between $\vec{H}$ and $\vec{N}$.

The diffuse component of the model [7] takes into account the surface roughness and is calculated according to the formula

$$\frac{k_d}{\pi} (1 + (F_{D90} - 1)(1 - \cos(\theta_i))^5) \cdot (1 + (F_{D90} - 1)(1 - \cos(\theta_o))^5), \quad (6.5)$$

where $F_{D90} = 0.5 + 2\alpha(\overline{H} \cdot \overline{L})^2$.

Multilayer materials, as well as fabrics and hair [5], are among the main materials that require the use of special physically-accurate BRDFs.

To model the reflective properties of fabrics, special microfacet and microcylindrical models are most often used. A common microfacet BRDF for fabrics is the Neubelt-Pettineo BRDF [5], which is built on the basis of the Ashikhmin-Premoze BRDF. A well-known microcylindrical BRDF for fabrics is the Kajiya-Kay BRDF [5], originally designed for hair. For hair, the basic BRDF is the Marschner BRDF [5], which takes into account the components R (reflection of light at the interface between air and hair fiber), TT (light from the air environment that passed through the hair fiber and went outside), TRT (light from the air environment that passed through the hair fiber, was reflected, passed through the fiber and went outside). In particular, the Huang model [8] was built on the basis of these components.

The basic models of light reflection for multilayer surfaces are the Kubelka-Munk and Hanrahan-Kruger models. However, these models do not have a closed mathematical form, so their use for image formation is limited. Simple transparent coating models are quite common, used to represent a thin transparent layer deposited on the surface of a base material. In particular, the Disney reflection model includes a specular lobe of the transparent layer if necessary. For multilayer surfaces, more complex and flexible reflection models are used, such as the Weidlich-Wilkie and Belcour BRDF [9]. The Weidlich-Wilkie model allows to simulate the reflection of light under an arbitrary number of environmental changes. Multiple changes in the environment are modeled by recursively calling the component BRDF. The Belcour model, unlike the Weidlich-Wilkie model, takes into account multiple light scattering. The model combines the sum of specular components based on the GGX distribution.

The disadvantage of this group of BRDFs is significant computational costs, therefore, physically-accurate BRDFs are rarely used in highly productive graphic systems.

Among the empirical models, characterized by high productivity and approximation accuracy of light reflection, the Blinn and Phong BRDFs [2,4,10] are the most frequently used.

The Phong and Blinn models for determining the specular color component have the form

$$\cos(x)^n, \quad (6.6)$$

where n is the shininess coefficient.

In the Phong distribution function (F_{PH}) the angle ψ between $\vec{V}, \vec{R}$ is used as the angle x , $\vec{R} = 2 \cdot (\vec{L} \cdot \vec{N}) \cdot \vec{N} - \vec{L}$. The vector $\vec{R}$ is called the specular reflection vector. In the Blinn distribution function (F_B), which historically appeared later than the Phong distribution function, the angle γ between $\vec{N}, \vec{H}$ is used instead of x (Fig. 6.1).

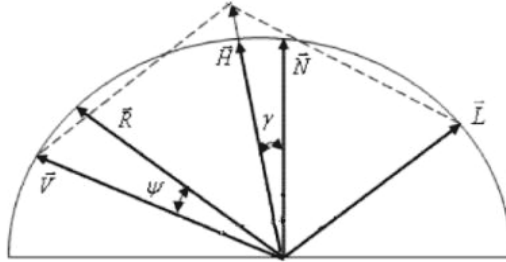


Fig. 6.1. Unit vectors to a surface point

When the n values are big, the Blinn and Phong BRDFs do not meet the requirements of highly productive graphics systems. Therefore, for the Blinn-Phong model the approximating BRDFs are used.

In modern three-dimensional graphics systems, the Schlick distribution function (F_{SH}) [11] has become widely used, which is calculated as

$$\cos(x) / (n - n \cos(x) + \cos(x)).$$

The Schlick BRDF has a significantly lower computational complexity compared to the Phong and Blinn functions. A significant disadvantage of the Schlick BRDF is that the distribution function in the attenuation zone falls to zero extremely slowly, which causes unnatural lighting of the graphic object and additional calculations due to an increase in the argument change interval. The low degree of the Schlick function makes it possible to use it in dynamic computer graphics tools, but the low accuracy of glare reproduction is an obstacle to its use for highly realistic reproduction of graphic scenes.

Another approximate expression is cosine-power BRDF [2]

$$\left(\frac{n}{a} (\cos(x) - 1) + 1 \right)^a,$$

where a — power of 2 number.

When $a = 16 (F_{16})$ the highly accurate approximation of Blinn BRDF is provided. When $a = 4 (F_4)$ or $a = 2 (F_2)$, the approximation is more productive. A compromise solution is to use the value $a = 8 (F_8)$.

The disadvantage of the cosine-power function is that its value increases after reaching zero. Therefore, additional computational costs are required for zeroing the increasing values.

The quadratic polynomial function (F_{kv}) [12] is calculated using the approximate formula

$$0.786n \cdot \cos(x)^2 + (1 - 0.786n) \cdot \cos(x).$$

The quadratic function reproduces the glare in the epicenter zone quite accurately but leads to an unnatural attenuation zone representation, because this function is a parabola and falls too quickly to the level of the ordinate axis zero. The other disadvantage of the function is the need to perform the clipping operation because of the abscissa axis crossing.

For the Blinn-Phong model approximation by a cubic function (F_{kub}) [12, 13] the calculated coefficients A, B, C , which depend on the shininess coefficient, are used. The cubic model of light reflection is calculated according to the formula

$$A \cos(x)^3 + B \cos(x)^2 + C \cos(x).$$

The values of the Q, G points of the ordinate axis are used to calculate the coefficients of the model. The point Q is usually located near the level of the inflection point of the function. The point G is located near the zero of the ordinate axis and is used to control the size of the glare's attenuation zone.

Coefficient $A F_{kUB}$ is calculated by the formula

$$\frac{LR(R-L) + GR(1-R) + LQ(L-1)}{L^3(R-R^2) + L^2(R^3-R) - LR^2(R-1)},$$

where $Q = \cos(t)^n = 0.5$, $G = \cos(u)^n = 1/18$, $R = \cos(t)$, $L = \cos(u)$.

Coefficient $B F_{kUB}$ is calculated according to the expression

$$\frac{GR(R^2-1) + LR(L^2-R^2) + QL(1-L^2)}{L^3(R-R^2) + L^2(R^3-R) - LR^2(R-1)}.$$

Coefficient $C F_{kUB}$ is calculated according to the formula

$$\frac{GR(R-R^2) + L^2Q(L-1) + LR(LR^2-L^2R)}{L^3(R-R^2) + L^2(R^3-R) - LR^2(R-1)}.$$

The disadvantage of F_{KUB} is an insufficiently accurate approximation of F_B in the attenuation zone, which leads to an unnatural reproduction of glare.

Empirical functions based on the calculation of the angle between vectors are characterized by increased complexity in determining the angle value, the ability to control the rate of light intensity attenuation. The requirement for this group of BRDFs is the usage of small powers k . The BRDFs based on the determination of the angle between vectors [14,15] for the approximation of the Blinn-Phong model are calculated by the formula [14,15]

$$\cos(a \cdot x)^k,$$

where k is the degree of the function (usually the smallest values are taken), a is the coefficient.

One of the types of BRDFs based on the determination of angle (denoted by F_{ang}) is calculated using the formula [14]

$$\cos\left(\sqrt{\frac{n}{k}} \cdot x\right)^k.$$

Other type of BRDFs based on determining the angle between vectors (denote $F_{kut\log}$) is calculated using the formula [15]

$$\cos^k\left(\frac{a \cos(e^{\frac{\ln Q}{k}})}{a \cos(\text{signum}(Q)^n \cdot |Q|^{\frac{1}{n}})} \cdot x\right),$$

where Q is the value of the Blinn-Phong BRDF, relative to which the coefficient of the argument is determined.

Such function can be further simplified. For example, when $Q = 0.5$, we can obtain the expression (F_{78})

$$\cos^k\left(\sqrt{\frac{7 \cdot n}{8 \cdot k}} \cdot x\right).$$

The logarithm function (F_{log}) based Blinn-Phong BRDF approximation [16] is characterized with low computational requirements and high precision glow's epicenter and attenuation zones reproduction. The function is calculated using the formula

$$(1 - 0.5 \cdot \log_2(1 - n \cdot \log_2(\cos(x)))).$$

For the specified above empirical models, in particular for F_{log} , $F_{kut\log}$, F_{ang} , the law of conservation of energy is not fulfilled. Regarding the BRDF,

the law of conservation of energy states that the light energy scattered by the surface cannot exceed the light energy incident on the surface.

Therefore, to ensure compliance with the law of conservation of energy, the models F_{PH} , F_B , F_2 , F_{16} are characterized by the calculated normalizing coefficients $(n+2)/2\pi$, $((n+2)(n+4))/(8\pi(2^{-n/2}+n))$, $(0.111/n+0.238\cdot(0.481+n))$, $(1.063n+1)/2\pi$ respectively.

Some empirical models are physically correct and are characterized by elements of physically accurate BRDF. For example, Gotanda BRDF (f_{GE}) [17] includes the calculation of the Fresnel coefficient

$$\frac{(n+2)}{4\pi(2-2^{-n/2})} \frac{F(F_0)\cos(\gamma)^n}{\max(\cos(\theta_i), \cos(\theta_o))},$$

where F_0 is the light reflectance at normal incidence.

Therefore, it is necessary to develop new BRDFs to increase the accuracy of glares reproduction and ensure highly productive rendering while maintaining a sufficient level of images quality.

6.3. DEVELOPMENT OF NEW BIDIRECTIONAL REFLECTANCE DISTRIBUTION FUNCTIONS

Let us consider a modification of Schlick BRDF [18]. We find a new bidirectional reflectance distribution function of the form

$$\frac{a_1 \cos(x)^{b_1}}{a_2 (a_3 n^{b_3} - a_4 (n * \cos(x))^{b_4} + a_5 \cos(x)^{b_5})^{b_2}}.$$

Let's find the unknown variables b_n (powers) and a_n (coefficients).

Because $\cos(x)$ is equal to 1 when $x=0$, therefore in the proposed Schlick BRDF $n - n\cos(x) = 0$. Hence $b_4 = b_3, a_4 = a_3$. Taking into account the latter, we obtain the following form of the proposed BRDF

$$\frac{a_1 \cos(x)^{b_1}}{a_2 (a_3 n^{b_3} - a_3 (n * \cos(x))^{b_3} + a_5 \cos(x)^{b_5})^{b_2}}.$$

We denote the proposed BRDF as F_{MSH} .

A program has been developed for selecting coefficients a_n and powers b_n , the flowchart of which is shown in Fig. 6.2.

To simplify the determination of unknown parameters in F_{MSH} we separate the procedures of coefficients and powers selection.

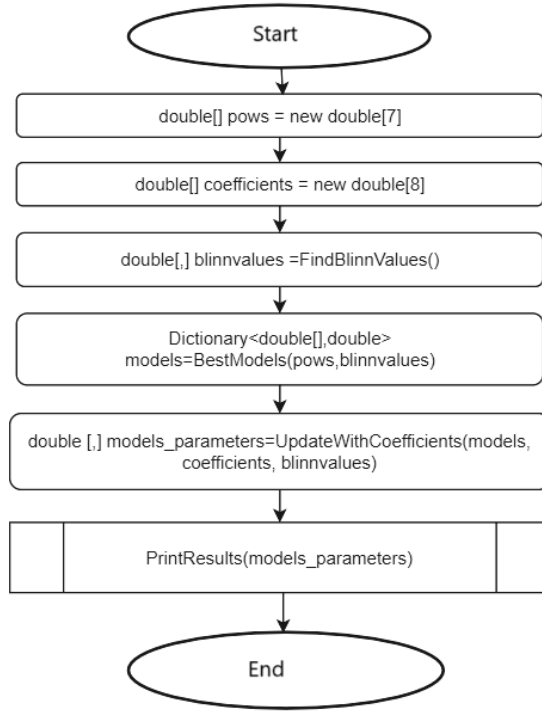


Fig. 6.2. Flowchart of F_{MSH} parameter selection

Let's choose the powers $b_1 - b_5$. At this stage, we take the values of the $a_1 - a_5$ coefficients as 1.

Considering the rapid growth of the computational complexity of the brute force method and the need for low computational complexity of the function, we will choose the value from the *pows* array of possible powers from the interval $[-3, 3]$.

The array of possible coefficients contains the values: 0.25, 0.5, 1, 1.25, 1.5, 2, 2.5, 3.

Further in a double cycle for angles $[0, 90]$, shininess coefficients n $[1, 1000]$ (inside the method *FindBlinnValues*) the function for calculating the Blinn BRDF values is called. The obtained values form a two-dimensional array *blinnvalues*.

In a four-dimensional loop inside the function *BestModels* (corresponding to 4 unknown powers) for n and angles, a two-dimensional array

of values F_{MSH} with random powers is filled. The average absolute deviation of the array from *blinnvalues* is calculated. Inside the loops, a list of five sets of powers with the smallest average absolute deviation is formed. The list is written to a dictionary *models* (the dictionary key is the set of powers, the value is the deviation).

In a four-dimensional (corresponding to 4 unknown coefficients) loop of the function *UpdateWithCoefficients*, random sets of coefficients are combined with the five found sets of powers. For each set of powers, a set of coefficients is determined that provides the smallest average absolute deviation from *blinnvalues*.

The function returns a two-dimensional array, where the rows represent information about the set of parameters F_{MSH} : the average absolute deviation from the Blinn BRDF, five power values, five coefficient values. The result of the function is saved to the *models_parameters* array.

PrintResults function outputs information about the obtained models (sets of the desired parameters).

Let us analyze the obtained results (Table 6.1). Model 1 has the highest accuracy. However, the model requires 5 raises to higher powers. Model 0 is the second most accurate. It requires 3 raises to higher powers. Model 2 is a compromise in accuracy and requires only one raise. Therefore, taking into account the accuracy and simplicity of the models, Model 2 was chosen.

Table 6.1

Obtained parameters for F_{MSH}

№	Degrees					Coefficients					Mean absolute deviation from Blinn BRDF
0	2	2	1	1	2	2	1.25	1	1	1.25	0.0063
1	3	2	1	1	3	2	1.25	1	1	1.25	0.0062
2	1	2	1	1	1	2	1.25	1	1	1.25	0.0066
3	2	2	1	1	3	2	1.25	1	1	1.25	0.0066
4	1	2	1	1	2	2	1.25	1	1	1.25	0.0070

For calculating the specular component, the most optimal is the following F_{MSH} form [18]:

$$\frac{2 \cos x}{\left(1 + \frac{1}{4}\right) \left(n - n^* \cos x + \left(1 + \frac{1}{4}\right) \cos x\right)^2}.$$

Let us analyze the proposed BRDF. We denote Schlick BRDF as F_{SH} , Blinn BRDF as F_B .

Fig. 6.3 shows the plots of F_{MSH} , F_{SH} , F_B . From the shown plots, it can be seen that the proposed F_{MSH} provides better reproduction of glare attenuation compared to F_{SH} , which makes it possible to eliminate its significant drawback — unnatural reproduction of the glare attenuation zone and excessive calculations.

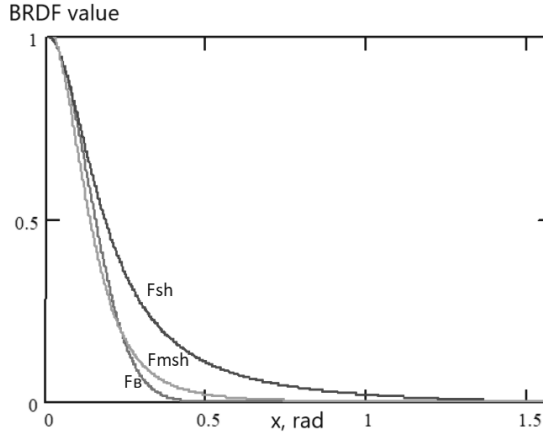


Fig. 6.3. Plots of F_{MSH} , F_{SH} , F_B ($n=60$)

Let us compare the relative errors of F_{MSH} and F_{SH} (δ_{msh} , δ_{sh} respectively) when approximating F_B at the point where the value $F_B = 0.3$. This level corresponds to the boundary of the epicenter zone of glare.

We find the abscissa of this point (x_{ep}) for the reference function F_B , which is equal to:

$$a \cos(\exp(\frac{-1.2}{n})).$$

Within n interval [2,1000] δ_{msh} is significantly smaller than δ_{sh} (no more than 16 % versus no more than 52 %) (Fig. 6.4).

The formula for calculating the absolute error during F_B approximation has the form

$$|F_{apr} - \cos(x)^n|,$$

where F_{apr} — F_B approximation

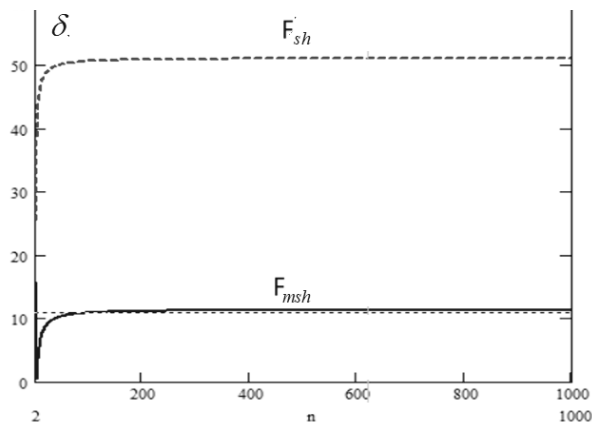


Fig. 6.4. δ_{msh} and δ_{sh} at point x_{ep}

Let's build three-dimensional plots (Fig. 6.5) of F_{MSH} , F_{SH} absolute errors from F_B ($\Delta_{F_{MSH}}$, $\Delta_{F_{SH}}$ respectively) depending on n and angle.

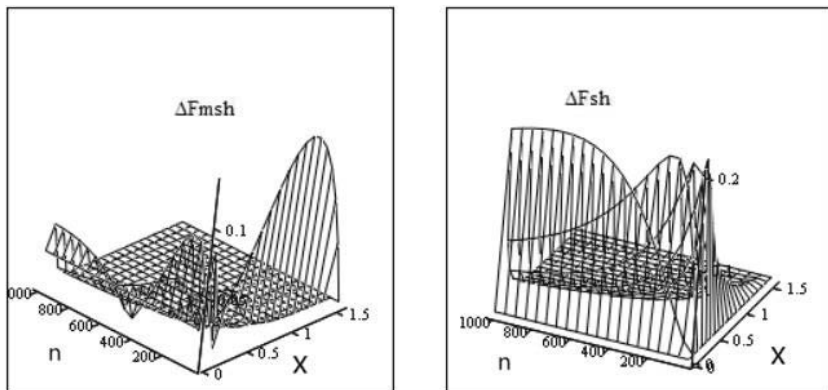


Fig. 6.5. Plots of $\Delta_{F_{MSH}}$, $\Delta_{F_{SH}}$ depending on n and the angle x

From the plots, we determine that the largest $\Delta_{F_{MSH}}$ is approximately half the largest $\Delta_{F_{SH}}$.

Let's find the values of the arguments at which F_{MSH} reaches zero. To do this, we equate F_{MSH} to zero. We find that F_{MSH} reaches zero at $x=1.570796$, which is approximately equal to $\frac{\pi}{2}$.

Next, we prove that the function F_{MSH} is monotonically decreasing. To do this, we find the derivative, which is equal to:

$$\frac{-1.6 * \sin(x)}{(n - n * \cos(x) + 1.25 \cos(x))^2} - \frac{3.2 * \cos(x)}{(n - n * \cos(x) + 1.25 * \cos(x))^3} (n \sin(x) - 1.25 \sin(x)) .$$

Fig. 6.6 shows a plot of the change in the derivative with respect to n and the angle x .

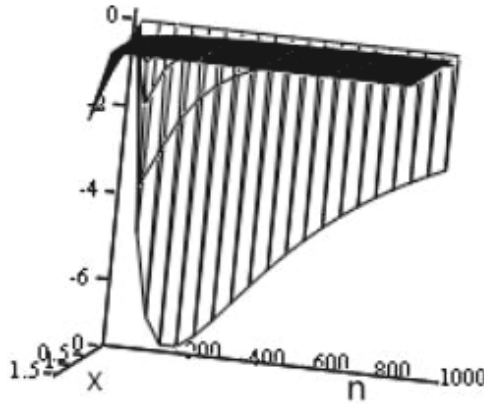


Fig. 6.6. Plot of the derivative of function

The function is monotonically decreasing because the value of its derivative is negative.

Let's calculate the ratio of the glare sizes for F_{MSH} and F_{SH} relative to F_B . We introduce a limit value 2^{-q} , which defines the argument beyond which the calculation of the BRDF stops.

First, we write an inequality for F_{SH} , which determines the interval for calculating the argument for determining the specular component of color.

$$0 \leq \frac{\cos x}{n - n \cos(x) + \cos(x)} \leq 2^{-q} .$$

As a result of solving the inequality, we obtain:

$$0 \leq x \leq a \cos\left(\frac{n}{2^q + n - 1}\right) .$$

Similarly, we write the inequality to determine the interval of change of the value for F_{MSH} :

$$0 \leq \frac{2 \cos x}{1.25(n - n \cos(x) + 1.25 \cos(x))^2} \leq 2^{-q}.$$

We find the value of the angle x :

$$0 \leq x \leq a \cos\left(\frac{4(16 * 2^q + 20n^2 - 25n - 4 * 2^{q+0.5} (8 + 20 * 2^{-q} n^2 - 25 * 2^{-q} n)^{\frac{1}{2}})}{5(16n^2 - 40n + 25)}\right).$$

For F_B the inequality looks like this

$$0 \leq \cos(x)^n \leq 2^{-q}.$$

The result of the solution is

$$0 \leq x \leq a \cos(2^{-\frac{q}{n}}).$$

We calculate $\Re$ (glare size ratio). For F_{SH} this value is calculated as follows

$$\frac{a \cos\left(\frac{n}{2^q + n - 1}\right)}{a \cos(2^{-\frac{q}{n}})}.$$

Next, we find $\Re$ for F_{MSH}

$$\frac{a \cos\left(\frac{4(16 * 2^q + 20n^2 - 25n - 4 * 2^{q+0.5} (8 + 20 * 2^{-q} n^2 - 25 * 2^{-q} n)^{\frac{1}{2}})}{5(16n^2 - 40n + 25)}\right)}{a \cos(2^{-\frac{q}{n}})}.$$

Fig. 6.7 shows plots of the ratios $\Re$ for F_{MSH} and F_B (A), F_{SH} and F_B (B).

Therefore, for the developed function the attenuation zone corresponds to an interval, which reduces the amount of calculations.

We will use the calculated limit values of BRDF arguments when constructing plots of maximum relative errors (δ) of F_{MSH} , F_{SH} depending on n .

The expression for determining δ of BRDF approximation relative to F_B is the

$$\left| \frac{100(F_{apr} - \cos(x)^n)}{\cos(x)^n} \right|.$$

To calculate the maximum δ of BRDF approximation for each n , taking into account the calculated limit values, a program was developed in MathCAD 2001. The program flowchart is shown in Fig. 6.8.

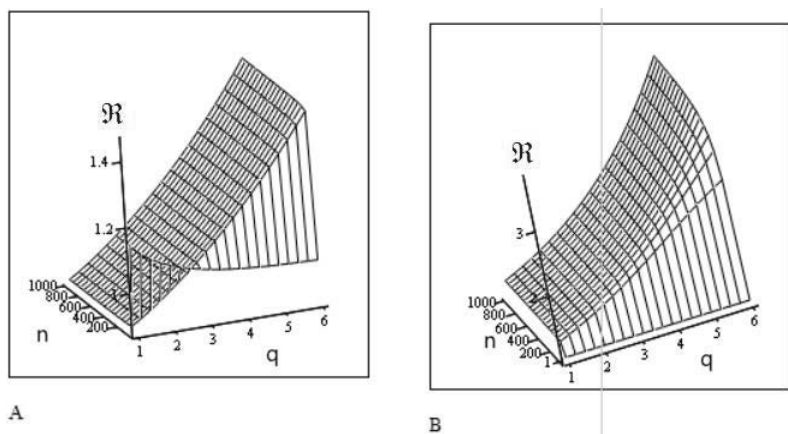


Fig. 6.7. Plots of ratios $\mathfrak{R}$ for F_{MSH} and F_B (A), F_{SH} and F_B (B)

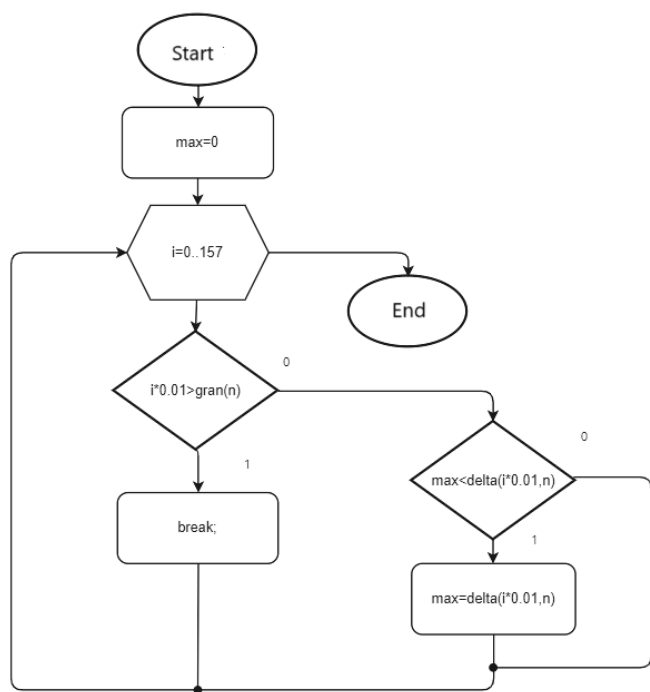


Fig. 6.8. Flowchart of the program for calculating the maximum δ for a given n

In the loop, the multiplier $i \cdot 0.01$, which corresponds to the angle value in radians, is compared with the determined limit value of BRDF calculation $gran(n)$. If the angle is greater, the loop is exited by the break operator. Otherwise, the temporary maximum value δ (variable *max*) is compared with the calculated value δ for the new angle (the result of the function *delta*). If the value *delta* is greater, the variable *max* is being updated.

Fig. 6.9 shows the constructed plots of the maximum δ of F_{MSH} , F_{SH} relative to n . The values for each n are calculated through a developed program.

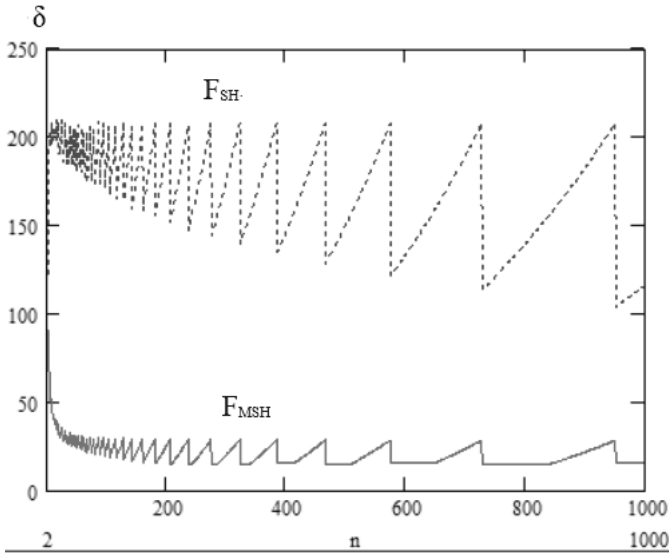


Fig. 6.9. Dependency of δ of F_{MSH} and F_{SH} from n

From the constructed plot, we conclude that maximum δ of F_{MSH} are less than the maximum δ of F_{SH} by 1.5–7 times for the sequence n .

Let us consider another approach to developing a modified Schlick function.

The approximate expression of the Blinn-Phong BRDF is based on the Schlick BRDF (F_{SH}). It is given in the form [19]

$$a \cdot F_{SH}^3 + b \cdot F_{SH}^2 + c \cdot F_{SH} + d,$$

where a, b, c, d — polynomial coefficients

We denote this expression as F_{bps} .

We create a dataset for selecting a, b, c, d values using the polynomial regression. To do this, we create a text file using the C # language, the first column X of which contains the values of the Schlick BRDF, the second column y — the values of the Blinn — Phong BRDF relative to $n \in [1, 1000]$ (step — 1), $x \in [0, 1.57]$ (step — 0.001 rad). An example of the generated dataset is shown in the Fig. 6.10.

```
0.9999500004166653,
0.9999395006100392,
0.9999280008639958,
0.9999155011900349,
0.9999020016006562,
0.9998875021093592,
0.9998720027306434,
0.9998555034800082,
0.9998380043739528,
0.9998195054299763,
```

Fig. 6.10. Dataset for coefficient selection

The selection of coefficients is carried out using the matrix equation of polynomial regression [20]

$$\begin{bmatrix} N & \sum_{i=0}^N X_i & \sum_{i=0}^N X_i^2 & \sum_{i=0}^N X_i^3 \\ \sum_{i=0}^N X_i & \sum_{i=0}^N X_i^2 & \sum_{i=0}^N X_i^3 & \sum_{i=0}^N X_i^4 \\ \sum_{i=0}^N X_i^2 & \sum_{i=0}^N X_i^3 & \sum_{i=0}^N X_i^4 & \sum_{i=0}^N X_i^5 \\ \sum_{i=0}^N X_i^3 & \sum_{i=0}^N X_i^4 & \sum_{i=0}^N X_i^5 & \sum_{i=0}^N X_i^6 \end{bmatrix} \cdot \begin{bmatrix} d \\ c \\ b \\ a \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^N y_i \\ \sum_{i=0}^N X_i y_i \\ \sum_{i=0}^N X_i^2 y_i \\ \sum_{i=0}^N X_i^3 y_i \end{bmatrix},$$

where N — the number of rows of the generated dataset, X_i — value F_{SH} , y_i — value F_B .

Having solved the equation, we find that $a = -1.631$, $b = 2.995$, $c = -0.374$, $d = 2.737 \cdot 10^{-3}$.

Therefore, F_{bps} it is calculated according to the expression [19]

$$\begin{aligned}
& -1.631 \cdot \left(\frac{\cos(x)}{n - n \cdot \cos(x) + \cos(x)} \right)^3 + 2.995 \cdot \left(\frac{\cos(x)}{n - n \cdot \cos(x) + \cos(x)} \right)^2 - \\
& -0.374 \cdot \frac{\cos(x)}{n - n \cdot \cos(x) + \cos(x)} + 2.737 \cdot 10^{-3}.
\end{aligned}$$

Fig. 6.11 shows the plots F_{bps} , F_{SH} , F_B at $n = 50$.

BRDF value

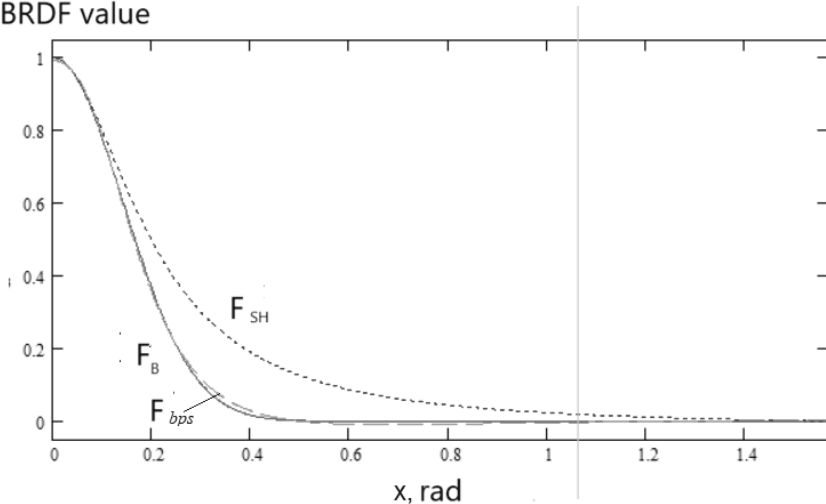


Fig. 6.11. Plots of F_{bps} , F_{SH} , F_B at $n = 50$

As can be seen from the figure, using F_{bps} for approximation of F_B allows to significantly increase the accuracy of reproducing the glare's attenuation zone compared to F_{SH} .

Fig. 6.12 shows a three-dimensional graph of F_{bps} values with respect to $n \in [1, 1000]$, $x \in [0, \pi/2]$.

Fig. 6.13 shows plots of maximum relative errors δ between F_B and F_{SH} , F_8 , F_{kv} , F_{kub} , F_{bps} in the epicenter zone of the glare for $n \in [8, 1000]$.

As shown in Fig. 13, F_{bps} over most of the interval $n \in [8, 1000]$ is characterized by the smallest maximum relative error of F_B approximation (1.05%) in the area of the glare epicenter compared to F_{SH} , F_8 , F_{kv} , F_{kub} .

Fig. 6.14 shows a three-dimensional plot of δ of F_{bps} from F_B relative to $n \in [8, 1000]$, $x \in [0, x_{per}]$ (x_{per} — the largest coordinate of the inflection point).

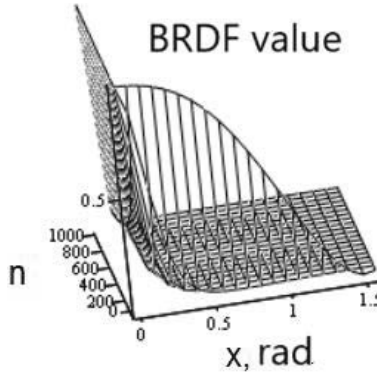


Fig. 6.12. Three-dimensional plot of F_{bps} values

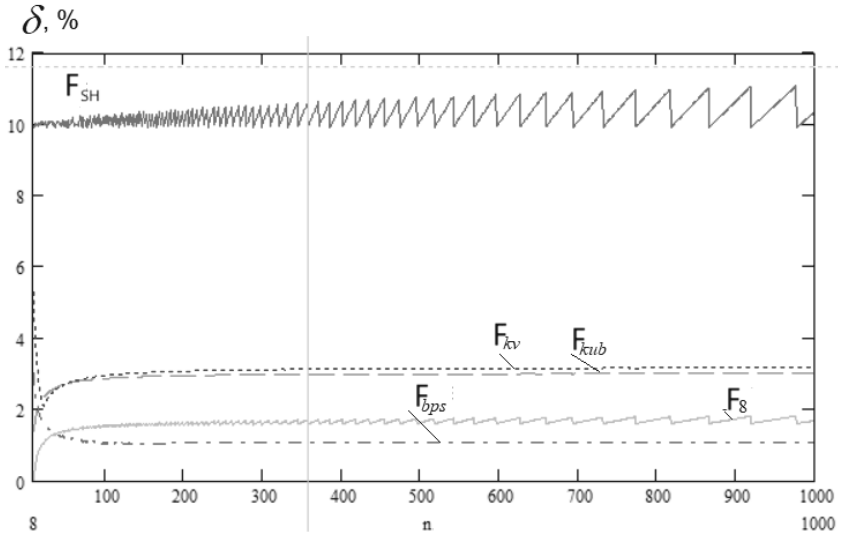


Fig. 6.13. Plots of maximum δ between F_B and F_{SH} , F_8 , F_{kv} , F_{kub} , F_{bps} in the area of the glare epicenter

Fig. 6.15 shows the plots of the maximum absolute errors Δ between F_B and F_{SH} , F_8 , F_{kv} , F_{kub} , F_{bps} for $n \in [8, 1000]$.

Therefore, F_{bps} is characterized by the smallest value of the maximum Δ from F_B (0.02) over most of the interval $n \in [8, 1000]$ compared to F_{SH} , F_8 , F_{kv} , F_{kub} .

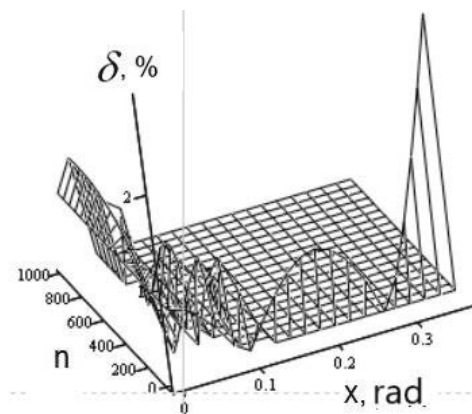


Fig. 6.14. Three-dimensional plot of δ between F_B and F_{bps} in the glare epicenter zone

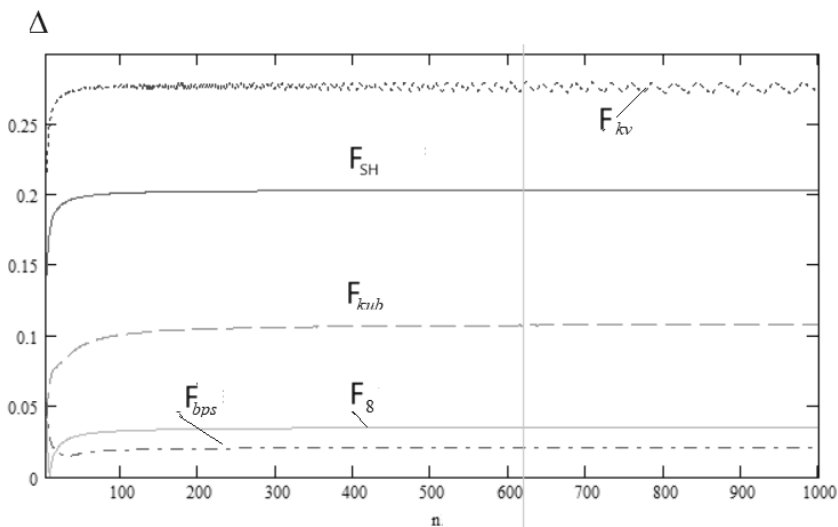


Fig. 6.15. Plots of maximum Δ between F_B and F_{SH} , F_8 , F_{kv} , F_{kub} , F_{bps}

Figure 6.16 shows a three-dimensional plot of Δ of F_{bps} from F_B relatively to $n \in [8, 1000]$, $x \in [0, \pi/2]$.

By taking the calculation expression of F_{SH} out of parentheses, we obtain a simpler calculation formula of F_{bps}

$$\left(\frac{\cos(x)}{n - \cos(x)(n-1)}\right)(-0.374 + \left(\frac{\cos(x)}{n - \cos(x)(n-1)}\right) \times \\ \times (2.995 - 1.631 \cdot \left(\frac{\cos(x)}{n - \cos(x)(n-1)}\right))) + 2.737 \cdot 10^{-3}.$$

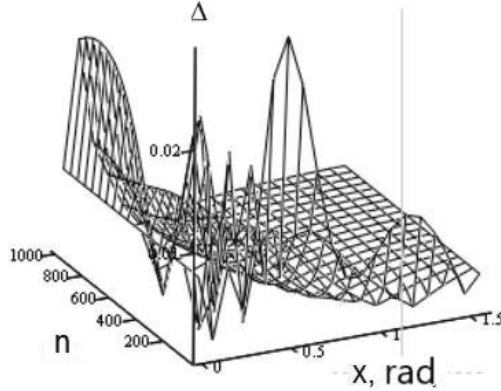


Fig. 6.16. Three-dimensional plot of Δ between F_B and F_{bps}

Let us compare the computational costs and accuracy of the applying the obtained formula F_{bps} with other Blinn-Phong approximation expressions in Table 6.2. The table contains the number of operations N_{op} of different types for calculating F_{SH} , F_8 , F_{kv} , F_{kub} , F_{bps} , corresponding total value of latency of AMD Zen 4 processor (2022) operations and maximum relative approximation errors from F_B in the glare epicenter zone. The considered lists of operations include operations for zeroing function values.

Table 6.2

Comparison of the number of operations for Blinn-Phong approximation expressions

Expres- sion	N_{op} multi- plication	N_{op} division	N_{op} addition	N_{op} incre- ment	N_{op} compar- ison	Total latency	δ , %
F_{SH}	1	1	2	0	0	15	11
F_8	4	1	0	2	0	24	1.8
F_{kv}	4	0	2	0	1	15	3
F_{kub}	5	1	2	0	0	27	3
F_{bps}	4	1	4	1	1	28	1.1

Compared to the original Schlick function F_{SH} , F_{bps} despite being twice as complex, it provides a 10-fold increase in the realism of glare epicenter reproduction on object surfaces.

Using the Idx3d software, a test figure “Dinosaur” was visualized based on F_{bps} . The visualization results are shown in Fig. 6.17.



Fig. 6.17. Example of forming a test figure “Dinosaur” based on F_{bps}

Therefore, using F_{bps} as an approximation function of F_B , it provides highly realistic visualization of three-dimensional objects in graphic scenes.

The next approach to modifying the Schlick BRDF is to use two functions: the Schlick BRDF and an additional Schlick BRDF with programmatically selected parameters. The value of the new BRDF is the average value of these functions.

The proposed BRDF based on two functions is calculated using the formula [21]

$$\frac{\cos(x)}{n - n * \cos(x) + \cos(x)} + \frac{\left(\frac{a_1 (\cos(x) + c_1)^{b_1}}{a_2 (a_3 n^{b_3} - a_4 (n * \cos(x))^{b_4} + a_3 \cos(x)^{b_5})^{b_2} + c_2} \right)^{b_6} - c_3}{2},$$

where a_n, b_n, c_n — are the unknown coefficients, powers, and addition terms, respectively.

C# program was used to select unknown coefficients, powers, and terms.

Table 6.3 presents the 5 most optimal sets of unknowns calculated by the program. The metric is the mean absolute deviation Δ from the Blinn BRDF values.

Table 6.3

Obtained sets of unknown values

Degrees ($b_1 - b_6$)						Coefficients ($a_1 - a_5$)					Addition terms ($c_1 - c_3$)			Δ
8	2	1	1	8	1	2	1.25	1.25	1.25	1.25	0.1	1.5	0.25	0.0025
2	1	1	1	4	2	1.5	0.5	3	3	3	0.25	0.5	0.25	0.0029
8	1	1	1	16	2	1.5	0.5	3	3	3	0.15	2.25	0.5	0.0033
4	1	1	1	8	2	1.25	0.5	2.5	2.5	2.5	0.2	1.1	0.25	0.0034
16	2	1	1	16	1	2	1.25	1.25	1.25	1.25	0	0	0.1	0.0074

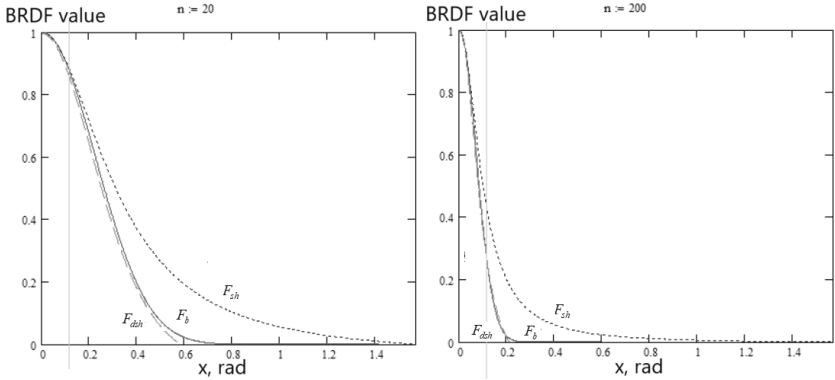
The first set of coefficients that provides the smallest absolute deviation is selected.

So, substituting the found coefficients into the formula, we obtain an expression for calculating the BRDF:

$$\frac{\cos(x)}{n - n * \cos(x) + \cos(x)} + \left(\frac{2(\cos(x) + 0.1)^8}{1.25(1.25n - 1.25(n * \cos(x)) + 1.25\cos(x)^8)^2 + 1.5} - 0.25 \right) \frac{1}{2}.$$

Let us denote this function as F_{DSH} .

Fig. 6.18 shows the plots F_{DSH} , F_{SH} , F_B for n 20, 200.

Fig. 6.18. Plots of F_{dsh} , F_{sh} , F_b for n 20, 200

The maximum relative error of the function from F_b in the epicenter zone of the glare is 5 % (Fig. 6.19).

The obtained BRDF more accurately approximates the Blinn-Phong BRDF compared to the Schlick BRDF, therefore, a more accurate reproduction of glare in the image of the object is ensured.

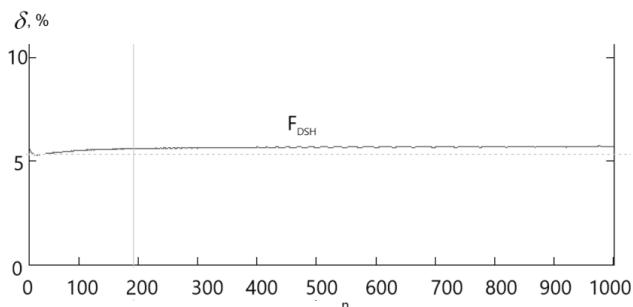


Fig. 6.19. Plots of relative error of F_{dsh} from F_b in the epicenter zone of the glare

Next, let's discuss the approximation of Blinn-Phong BRDF based on combined BRDFs.

The main idea of the first method lies in using low-degree BRDF for the glare epicenter and using high-degree BRDF for the glare attenuation zone. The combined reflectance model is based on cosine-power functions (let's denote the function as F_{416}) and is calculated using the formulas [22]

$$\begin{cases} F_4, & x \leq \text{merge}(n) \\ F_{16}, & \text{merge}(n) < x \leq \text{lim}(n), \\ 0, & x > \text{lim}(n) \end{cases}$$

where $\text{merge}(n)$ — the point of cosine-power functions connection; $\text{lim}(n)$ — the point, where the value of combined function equals 0,

$$F_4 = \frac{n}{4}(\cos(x) - 1 + 1)^4, \quad F_{16} = \left(\frac{n}{16}(\cos(x) - 1 + 1)\right)^{16}.$$

$\text{lim}(n)$ was introduced because after reaching 0 the values of F_{16} begin increasing. $\text{lim}(n)$ is calculated using the formula

$$a \cos\left(\frac{n-16}{n}\right).$$

This formula can be approximated using the expression

$$\frac{6}{\sqrt{n}}.$$

Let us find the formula for $\text{merge}(n)$ point, where $F_4 = F_{16}$. One of the requirements to $\text{merge}(n)$ is its belonging to the interval $x \in (0, t_{f4=0})$, where $t_{f4=0}$ — the point at which $F_4 = 0$. MathCAD analysis has shown that for the interval $x \in (0, t_{f4=0})$ the point where $F_4 = F_{16}$ doesn't exist.

Therefore, for the $merge(n)$ we chose the point where F_{16} and F_4 values differ by $\frac{1}{128}$ (this value was chosen experimentally). We solve the equation

$$\left| \left(\frac{n}{4} (\cos(x) - 1) + 1 \right)^4 - \left(\frac{n}{16} (\cos(x) - 1) + 1 \right)^{16} \right| = \frac{1}{128}.$$

From the equation we find that $merge(n)$ is calculated using the formula [22]

$$a \cos\left(\frac{n - 0.33}{n}\right).$$

For the calculation simplification, $merge(n)$ was approximated with the piecewise function. $merge(n)$ graph was divided into four intervals $n \in [1, 8], n \in (8, 30], n \in (30, 150], n \in (150, 1000]$. At the first interval $merge(n)$ was approximated with the third-degree polynomial of the form $An^3 + Bn^2 + Cn + D$. At other intervals $merge(n)$ was approximated with the first-degree polynomials of the form $An + B$.

For the approximation the Chebyshev polynomials were used, they are calculated using the formula [22]

$$f(x) \approx \left[\sum_{k=0}^{m-1} c_k T_k \left(\frac{x - 0.5(b+a)}{0.5(b-a)} \right) \right] - \frac{c_0}{2},$$

where

$$c_j = \frac{2}{m} \sum_{k=1}^m f \left[\cos \left(\frac{\pi(k-0.5)}{m} \right) \cdot 0.5(b-a) + 0.5(b+a) \right] \cos \left(\frac{\pi \cdot j \cdot (k-0.5)}{m} \right),$$

$T_0(x) = 1$, $T_1(x) = x$, $T_k(x) = \cos(k \cdot \arccos(x))$, $[a, b]$ — interval of approximation, $m-1$ — polynomial degree.

Then $merge(n)$ is calculated using the formula

$$\begin{cases} -0.0039n^3 + 0.0663n^2 - 0.3934n + 1.16, n \in [1, 8] \\ -5.526 \cdot 10^{-3} \cdot n + 0.305, n \in (8, 30] \\ -5.567 \cdot 10^{-4} \cdot n + 0.144, n \in (30, 150] \\ -3.591 \cdot 10^{-5} \cdot n + 0.059, n \in (150, 1000] \end{cases}.$$

The values of $merge(n)$ at the interval $n \in [1, 8]$ are intended to be read from graphics system memory. We simplify the formulas for other intervals by representing fractional values as the numbers of power two. The updated $merge(n)$ formula is defined as

$$\left\{ \begin{array}{l} -0.0039n^3 + 0.0663n^2 - 0.3934n + 1.16, n \in [1, 8] \\ -\left(\frac{1}{2^8} + \frac{1}{2^9}\right) \cdot n + \frac{1}{2^2} + \frac{1}{2^4}, n \in (8, 30] \\ -\left(\frac{1}{2^{11}} + \frac{1}{2^{14}}\right) \cdot n + \frac{1}{2^3} + \frac{1}{2^6}, n \in (30, 150] \\ -\left(\frac{1}{2^{15}} + \frac{1}{2^{18}}\right) \cdot n + \frac{1}{2^4} - \frac{1}{2^8}, n \in (150, 1000] \end{array} \right.$$

Fig. 6.20 shows the plots of original (*merge*) and approximated (*ap_merge*) *merge(n)* functions at the interval $n \in [1, 1000]$.

Function value

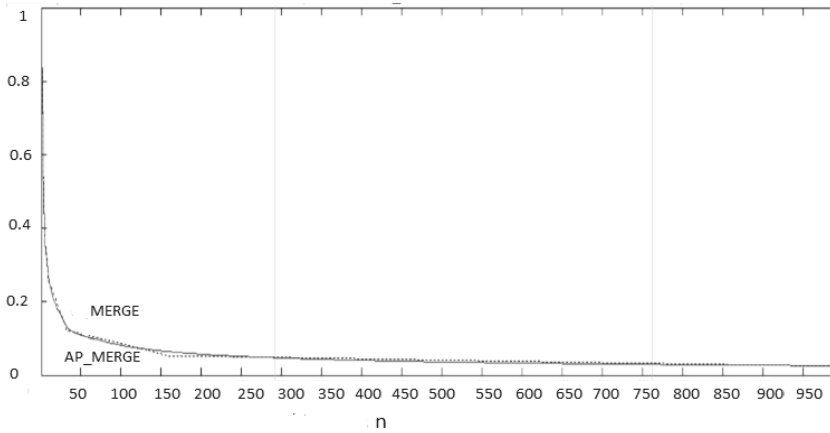


Fig. 6.20. The plots of original and approximated formulas of cosine-power functions connection point with respect to shininess

Let's compare the values of derivatives of F_{16} (Der_{16}) and F_4 (Der_4) at the point *merge(n)* for $n \in [1, 1000]$ (Fig. 6.21).

As shown in Fig. 6.21, the values of Der_{16} and Der_4 at the point *merge(n)* are approximately equal respective to $n \in [1, 1000]$. Therefore, the smooth connection of F_{16} and F_4 functions is provided.

Fig. 6.22 shows the plot of absolute deviation between F_{16} and F_4 near the point *merge(n)* when $n = 50, 150$. The insignificant deviation between F_{16} and F_4 near *merge(n)* shows that the connection of functions is smooth.

Fig. 6.23 shows the plots of F_{416} and F_B when $n = 120$ and $x \in [0, 0.55]$.

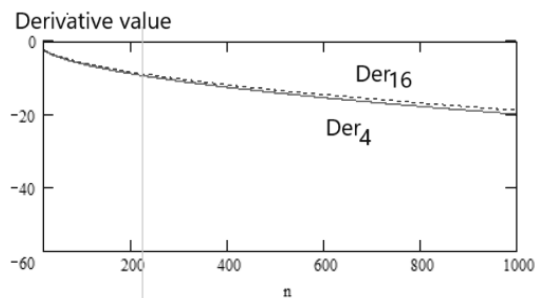


Fig. 6.21. Values of F_{16} and F_4 derivatives

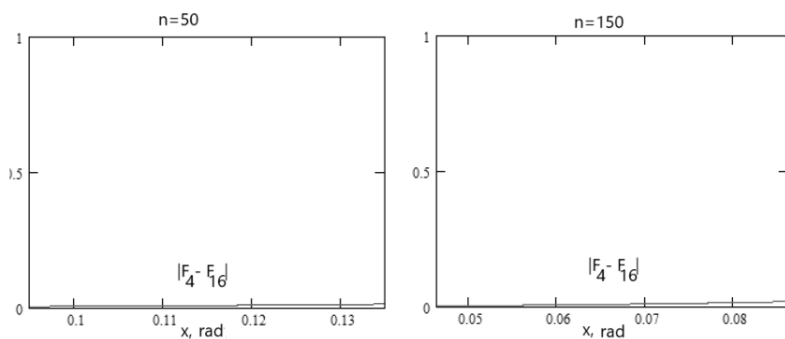


Fig. 6.22. Deviation between F_{16} and F_4 near $merge(n)$ point

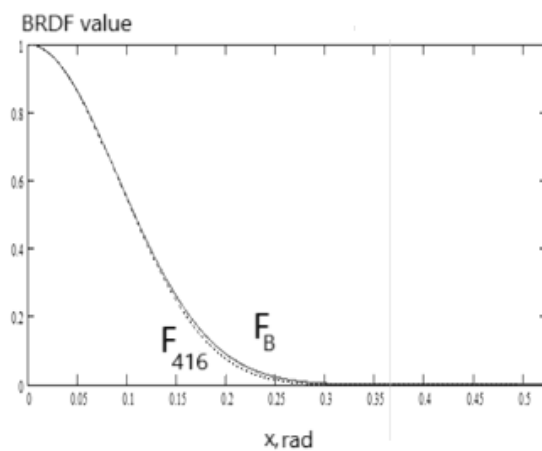


Fig. 6.23. The plots of F_{416} and F_B BRDFs with respect to angle values x

Let us analyze the efficiency gain from the F_{416} calculation. The plot in Fig. 6.24 shows what percentage of the F_{416} curve length is calculated as F_4 for $n \in [1, 1000]$.

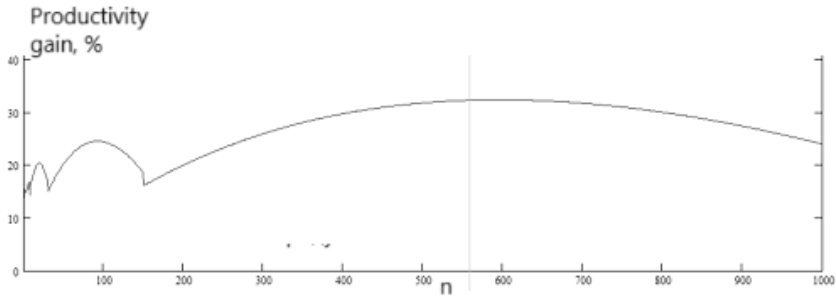


Fig. 6.24. The plot of efficiency gain from the combined function calculation with respect to shininess n

The mean value of efficiency gain is 27.3 %.

Fig. 6.25 shows the plots of maximum relative errors δ between F_{416} , F_{16} , F_4 and F_B in the glare's epicenter zone for $n \in [4, 1000]$.

δ , %

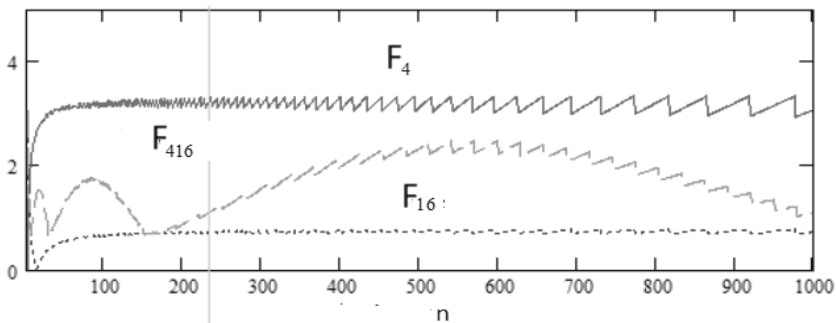


Fig. 6.25. Maximum relative errors between F_{416} , F_{16} , F_4 and F_B in the glare's epicenter zone with respect to n

The maximum δ of F_4 from F_B at the interval $n \in [4, 1000]$ is 3.3 %. The maximum δ of F_{416} from F_B at the majority of the interval $n \in [4, 1000]$ is 2.4 %. Therefore, F_{416} is more precise than F_4 and is quicker to calculate than F_{16} .

Fig. 6.26 shows the plots of maximum absolute errors Δ between F_{416} , F_{16} , F_4 and F_B for $n \in [4, 1000]$. Fig. 6.27 shows the plot of absolute error Δ between F_{416} and F_B with respect to $n \in [4, 1000]$ and $x \in [0; \pi/2]$.

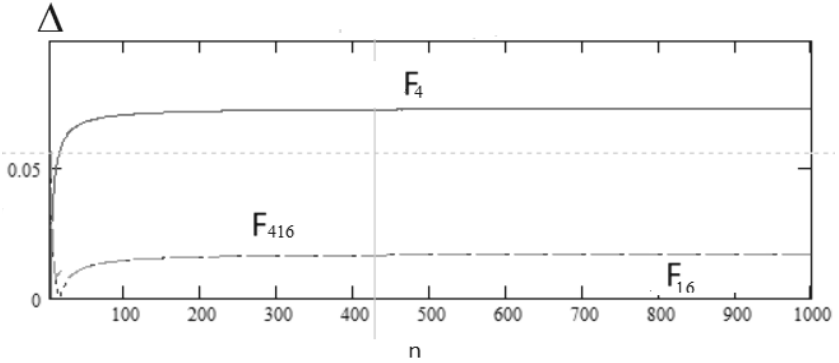


Fig. 6.26. Maximum absolute errors between F_{416} , F_{16} , F_4 and F_B with respect to shininess n

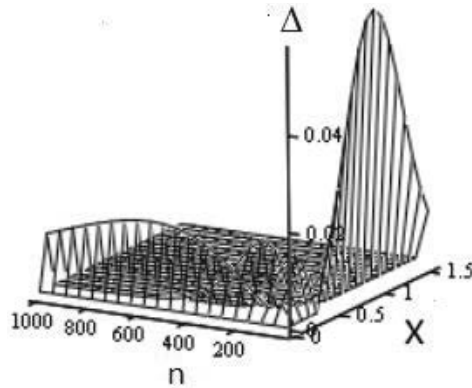


Fig. 6.27. Absolute errors between F_{416} and F_B with respect to shininess n and angle x

The maximum Δ of F_4 from F_B at the interval $n \in [4, 1000]$ is 0.07.

The maximum Δ of F_{416} from F_B at the majority of the interval $n \in [4, 1000]$ is 0.02.

In the software product BRDF Explorer the teapot was visualized using OpenGL shading language and the proposed F_{416} .

The result of the teapot visualization is shown in Fig. 6.28.



Fig. 6.28. The result of teapot visualization using F_{416}

Therefore, the developed bidirectional reflectance distribution function provides highly-productive and highly-realistic object's glare reproduction.

The second method lies in F_{KV} calculation in the epicenter zone of glare and calculation of modified F_{KUB} (F_{KUB2}) in the attenuation zone. The combined function is defined as [12]

$$\begin{cases} F_{KV}, x \leq \text{connect}(n) \\ F_{KUB2}, \text{connect}(n) < x \end{cases}$$

where $F_{KV} = 0.786n \cdot \cos(x)^2 + (1 - 0.786n) \cdot \cos(x)$, $\text{connect}(n)$ — the connection point of F_{KV} and F_{KUB2} .

Let us find the modified cubic function F_{KUB2} , that will provide the F_B approximation accuracy increasing in the attenuation zone compared to the F_{KUB} . In order to do this we create the system of equations

$$\begin{cases} A2 \cdot \cos(E_1)^3 + B2 \cdot \cos(E_1)^2 + C2 \cdot \cos(E_1) + D2 = 0.6, \\ A2 \cdot \cos(t)^3 + B2 \cdot \cos(t)^2 + C2 \cdot \cos(t) + D2 = Q, \\ A2 \cdot \cos(u)^3 + B2 \cdot \cos(u)^2 + C2 \cdot \cos(u) + D2 = G, \\ D2 = 0 \end{cases}$$

where $E_1 = a \cos(e^{-0.511/n})$, 0.6 — the ordinate axis level of F_{KV} and F_{KUB} connection

From the system we find that the coefficient F_{KUB2} $A2$ is calculated using the formula [12]

$$0.2 \frac{LR(3R-3L)+GR(5E^2-R5E)+LQ(L5E-5E^2)}{L^3(RE^2-R^2E)+L^2(R^3E-RE^3)-LR^2(RE^2-E^3)},$$

where $E = \cos(E_1) = e^{-0.511/n}$, $Q = \cos(t)^n = 0.125$, $G = \cos(u)^n = 1/32$, $R = \cos(t)$, $L = \cos(u)$.

Coefficient $B2$ is calculated using the formula [12]

$$0.2 \frac{GR(R^25E-5E^3)+LR(3L^2-3R^2)+QL(5E^3-L^25E)}{L^3(RE^2-R^2E)+L^2(R^3E-RE^3)-LR^2(RE^2-E^3)}.$$

Coefficient $C2$ is calculated using the formula [12]

$$0.2 \frac{GR(R5E^3-R^25E^2)+L^2Q(L5E^2-5E^3)+LR(3LR^2-3L^2R)}{L^3(RE^2-R^2E)+L^2(R^3E-RE^3)-LR^2(RE^2-E^3)}.$$

Using the MS Excel tools the simplified polynomial formulas of these coefficients are obtained. Coefficient $A2$ is computed using the expression

$$\left(\frac{1}{2^4} + \frac{1}{2^6} + \frac{1}{2^{10}} + \frac{1}{2^{11}} + \frac{1}{2^{16}}\right)n^2 + \left(\frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^7} + \frac{1}{2^8}\right)n + \left(\frac{1}{2^7} + \frac{1}{2^8} - \frac{1}{2^{11}}\right).$$

The simplified formula of $B2$ is defined as

$$\left\{ \begin{array}{l} -\left(\frac{1}{2^{16}} - \frac{1}{2^{20}}\right)n^3 - \left(\frac{1}{2^3} + \frac{1}{2^5} + \frac{1}{2^9} + \frac{1}{2^{14}}\right)n^2 + \\ + \left(\frac{1}{2^3} - \frac{1}{2^5} - \frac{1}{2^9}\right)n + \left(\frac{1}{2^1} + \frac{3}{2^3} - \frac{1}{2^5} - \frac{1}{2^9}\right) \text{ if } n \geq 5 \wedge n < 32 \\ -\left(\frac{1}{2^3} + \frac{1}{2^5} + \frac{1}{2^9} + \frac{1}{2^{10}} + \frac{1}{2^{16}}\right)n^2 + \left(\frac{1}{2^3} - \frac{1}{2^7} - \frac{1}{2^8} - \frac{1}{2^{10}} - \frac{1}{2^{11}}\right)n + \\ + \left(\frac{1}{2^1} + \frac{1}{2^3} + \frac{1}{2^5} - \frac{1}{2^9}\right) \text{ if } n \geq 32 \wedge n \leq 256 \end{array} \right.$$

The simplified formula of $C2$ is lies in calculating the expression

$$\left(\frac{1}{2^4} + \frac{1}{2^6} + \frac{1}{2^{10}} + \frac{1}{2^{11}}\right)n^2 - \left(\frac{1}{2^2} + \frac{1}{2^4} - \frac{1}{2^{10}} - \frac{1}{2^{11}}\right)n + \left(\frac{1}{2^3} + \frac{1}{2^4} - \frac{1}{2^7} - \frac{1}{2^9}\right).$$

Fig. 6.29 shows the plots of F_{KUB2} , F_{KV} , F_B .

Therefore, the connection of F_{KUB2} and F_{KV} provides the highly accurate F_B approximation.

Let us find the F_{KUB2} and F_{KV} connection point. We solve the equation

$$\begin{aligned}
& 0.786n \cdot \cos(x)^2 + (1 - 0.786n) \cdot \cos(x) = \\
& = A2 \cdot \cos(x)^3 + B2 \cdot \cos(x)^2 + C2 \cdot \cos(x) .
\end{aligned}$$

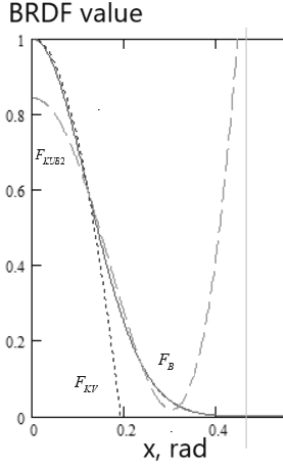


Fig. 6.29. Plots of F_{KUB2} , F_{KV} , F_B when $n = 70$

The precise formula for the calculation of connection point $connect(n)$ between F_{KUB2} and F_{KV} when $n \geq 32$ is defined as

$$\begin{aligned}
& 1304125n^2 - 5360000 + 5522912n - \\
& a \cos\left(\frac{-\sqrt{15625n^4 + 8453383423744n^2 - 564728000n^3 + 31208448000000 - 16664864256000n}}{2(652125n^2 + 1632000n + 92000)}\right) .
\end{aligned}$$

The expression is calculated using the formula when $n < 32$

$$\begin{aligned}
& a \cos\left(\frac{-110336 \cdot 10^3 + 1875n^3 + 20744 \cdot 10^3 n^2 + 90990592n}{2(26112000n + 10434000n^2 + 1472000)} - \right. \\
& \left. - \sqrt{\frac{12808617984 \cdot 10^6 + 86744733184 \cdot 10^3 n^3 + 1636015759294464n^2 - 918864330752 \cdot 10^4 n + 3515625n^6 + 7779 \cdot 10^7 n^5 - 473520128 \cdot 10^4 n^4}{2(26112000n + 10434000n^2 + 1472000)}}\right) .
\end{aligned}$$

The simplified expression for $connect(n)$ is computed using the formula

$$\begin{cases} (\frac{1}{2^{12}} + \frac{1}{2^{18}})n^2 - \frac{1}{2^6}n + (\frac{1}{2^1} - \frac{1}{2^4} - \frac{1}{2^6} - \frac{1}{2^7} - \frac{1}{2^9}) & \text{if } n < 32 \\ (\frac{1}{2^{16}} - \frac{1}{2^{20}} - \frac{1}{2^{21}})n^2 - (\frac{1}{2^7} - \frac{1}{2^8} - \frac{1}{2^{10}} + \frac{1}{2^{15}})n + (\frac{1}{2^2} + \frac{1}{2^7} + \frac{1}{2^{10}}) & \text{if } n \geq 32 \wedge n < 100 \\ (\frac{1}{2^{20}})n^2 - (\frac{1}{2^{11}} + \frac{1}{2^{14}} + \frac{1}{2^{15}})n + (\frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^7}) & \text{if } n \geq 100 \end{cases}$$

At the ordinate axis level 0.6 the smooth connection of function is provided because of the difference of values between F_{KUB2} and F_{KV} near $connect(n)$ is insignificant (Fig. 6.30). Consequently, the derivatives calculation is not needed.

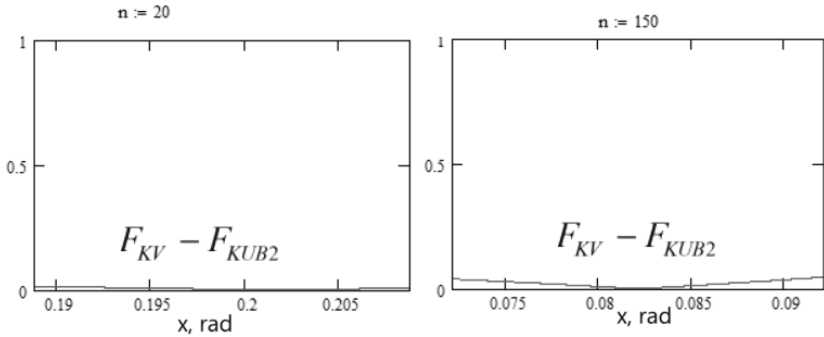


Fig. 6.30. The deviations between F_{KV} and F_{KUB2} near $connect(n)$

Let us denote the developed combined function as F_{COMB} . Fig. 6.31 shows the plots of F_{COMB} , F_{KUB} , F_{KV} , F_B when $n = 36$.

Therefore, F_{COMB} provides the increasing of attenuation zone reproduction accuracy while speeding up the glare's epicenter zone calculation compared to F_{KUB} .

On average, the 40.2% of F_{COMB} curve length is calculated as F_{KV} (Fig. 6.32), this value is a productivity gain.

Fig. 6.33 shows the plots of maximum relative errors δ between F_{COMB} , F_{KUB} , F_{KV} and F_B in the glare's epicenter for $n \in [5, 256]$.

Over the greater part of the interval $n \in [5, 256]$ the maximum relative errors of the specified functions coincide and are equal to 3.04%.

Fig. 6.34 shows the plots of maximum absolute errors Δ between F_{COMB} , F_{KUB} , F_{KV} and F_B for $n \in [5, 256]$.

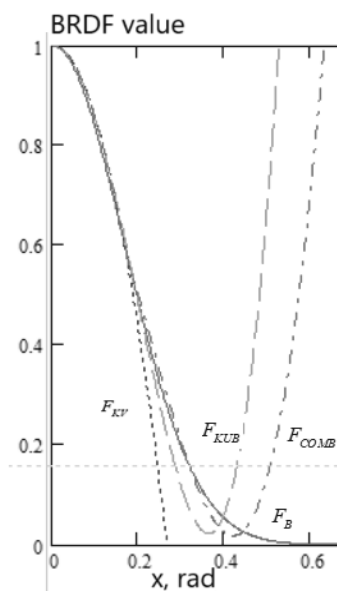


Fig. 6.31. The plots of F_{COMB} , F_{KUB} , F_{KV} , F_B when $n=36$

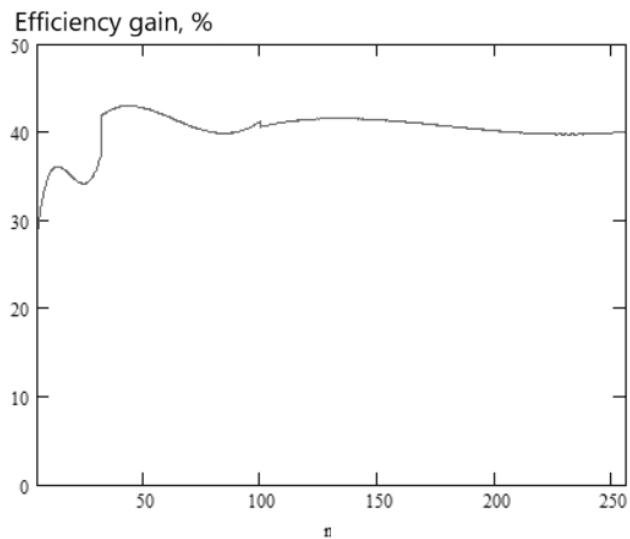


Fig. 6.32. The plot of productivity gain from F_{COMB} calculation

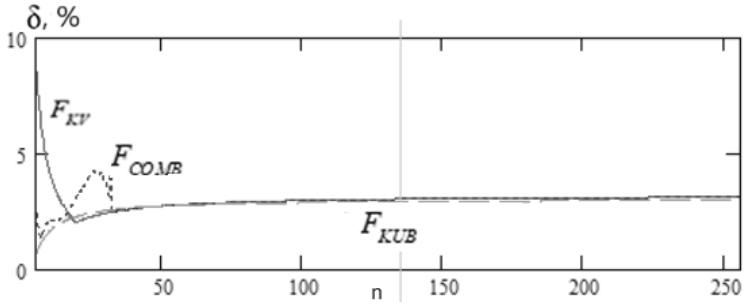


Fig. 6.33. Plots of maximum relative errors between F_{COMB} , F_{KUB} , F_{KV} and F_B in the epicenter zone

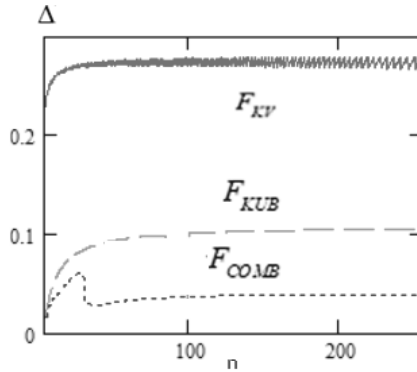


Fig. 6.34. The plots of maximum absolute errors between F_{COMB} , F_{KUB} , F_{KV} and F_B

The maximum ΔF_{KUB} is 0.1, maximum ΔF_{KV} — 0.28, maximum ΔF_{COMB} — 0.06.

Fig. 6.35 shows the results of applying F_{COMB} , F_{KUB} , F_{KV} to the visualization of the figures “Teapot”, “Robot” in BRDF Explorer.

Therefore, F_{COMB} is characterized with the increased calculation productivity and more realistic glare’s attenuation zone reproduction.

The third method of combined BRDF calculation lies in the following.

To ensure a highly accurate F_B approximation, we combine F_{KUB} with another cubic function F_{KUB22} , which will allow us to more accurately reproduce the attenuation zone of the glare. The combined function is defined as [13]

$$\begin{cases} F_{KUB}, x \leq \text{connect}(n) \\ F_{KUB22}, \text{connect}(n) < x, \end{cases}$$

where $\text{connect}(n)$ — connection point between F_{KUB} and F_{KUB22} .

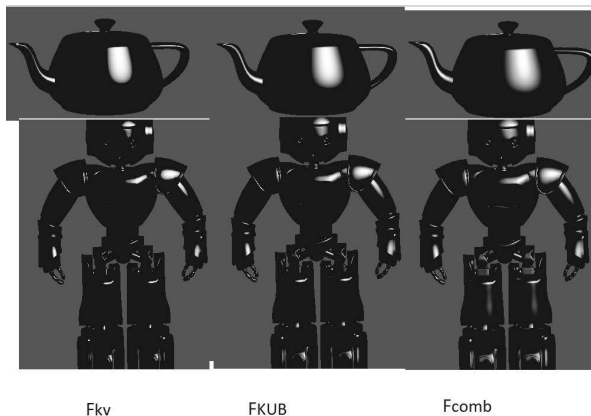


Fig. 6.35. The results of figures visualization based on F_{COMB} , F_{KUB} , F_{KV}

We denote the developed combined function as $F_{KUBCOMB}$.

For the connection of F_{KUB} with the new cubic function F_{KUB22} , we will choose the level of the ordinate axis 0.5, below which the values of F_{KUB} and F_B are noticeably different.

Let's determine the coefficient formulas for F_{KUB22} , using a system of equations. The first equation corresponds to the level $\cos(E_1)^n = 0.5$, the second equation corresponds to the level $\cos(t)^n = Q$, the third — to the level $\cos(u)^n = G$, the fourth — to the level $\cos(\pi/2)^n = 0$. The system of equations is defined as

$$\begin{cases} A2 \cdot \cos(E_1)^3 + B2 \cdot \cos(E_1)^2 + C2 \cdot \cos(E_1) + D2 = 0.5, \\ A2 \cdot \cos(t)^3 + B2 \cdot \cos(t)^2 + C2 \cdot \cos(t) + D2 = Q, \\ A2 \cdot \cos(u)^3 + B2 \cdot \cos(u)^2 + C2 \cdot \cos(u) + D2 = G, \\ D2 = 0 \end{cases}$$

where $E_1 = a \cos(e^{-0.693/n})$.

From the system we find that the coefficient $A2$ is calculated according to the formula [13]

$$0.5 \frac{LR(R-L) + GR(2E^2 - R^2E) + LQ(L^2E - 2E^2)}{L^3(RE^2 - R^2E) + L^2(R^3E - RE^3) - LR^2(RE^2 - E^3)},$$

where $E = \cos(E_1) = e^{-0.693/n}$, $Q = \cos(t)^n = 0.1$, $G = \cos(u)^n = 1/40$, $R = \cos(t)$, $L = \cos(u)$.

$B2$ is calculated according to the formula [13]

$$0.5 \frac{GR(R^2 2E - 2E^3) + LR(L^2 - R^2) + QL(2E^3 - L^2 2E)}{L^3(RE^2 - R^2E) + L^2(R^3E - RE^3) - LR^2(RE^2 - E^3)}.$$

$C2$ is calculated according to the formula [13]

$$0.5 \frac{GR(R^2 2E^3 - R^2 2E^2) + L^2 Q(L^2 E^2 - 2E^3) + LR(LR^2 - L^2 R)}{L^3(RE^2 - R^2E) + L^2(R^3E - RE^3) - LR^2(RE^2 - E^3)}.$$

Fig. 6.36 shows the plots of F_{KUB22} , $A2$, $B2$, $C2$ coefficient values when $Q = 0.1$, $G = 1/40$.

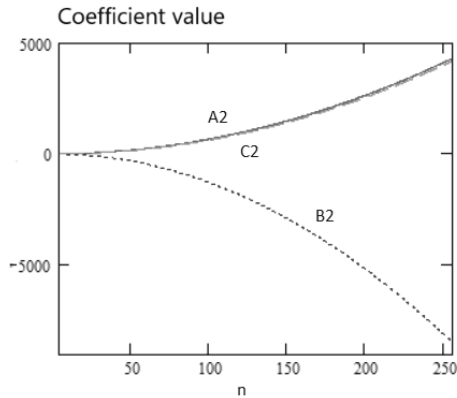


Fig. 6.36. The plots of coefficient $A2$, $B2$, $C2$ values

It is advisable to store the calculated F_{KUB} and F_{KUB22} coefficients in a block of permanent memory, since they are the same for all cases of calculation of BRDF.

Fig. 6.37 shows the plots of F_{KUB22} , F_{KUB} and F_B when $n = 50$. The selected values for Q, G , F_{KUB22} are 0.1, 1/40.

As can be seen from the figure, the combination of F_{KUB22} , F_{KUB} at the point 0.5 provides a highly accurate approximation of F_B in the epicenter and attenuation zones.

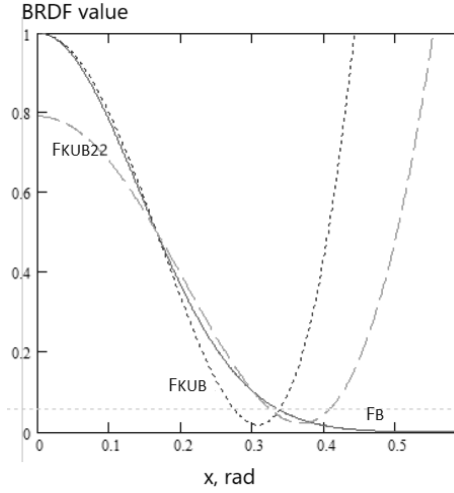


Fig. 6.37. Plots of F_{KUB22} , F_{KUB} and F_B when $n=50$

Let's find the abscissa of the connection point of F_{KUB} and F_{KUB22} . We equate the calculation formulas of F_{KUB} and F_{KUB22}

$$A \cdot \cos(x)^3 + B \cdot \cos(x)^2 + C \cdot \cos(x) = A2 \cdot \cos(x)^3 + B2 \cdot \cos(x)^2 + C2 \cdot \cos(x).$$

From the equation, we find the formula for the point of connection of F_{KUB} and F_{KUB22} $connect(n)$ [13]

$$connect(n) = a \cos(e^{-0.693/n}).$$

With the usage of MS Excel, the simplified calculation expressions of $connect(n)$ ($connect_appr(n)$) depending on n were obtained

$$\begin{cases} (\frac{1}{2^9} - \frac{1}{2^{12}})n^2 - (\frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7})n + (\frac{1}{2^1} + \frac{1}{2^2}), n \geq 4 \wedge n \leq 16 \\ (\frac{1}{2^{15}} + \frac{1}{2^{16}} + \frac{1}{2^{18}})n^2 - (\frac{1}{2^7} - \frac{1}{2^{10}})n + (\frac{1}{2^2} + \frac{1}{2^3}), n > 16 \wedge n \leq 70 \\ (\frac{1}{2^{19}} - \frac{1}{2^{22}})n^2 - (\frac{1}{2^{10}} - \frac{1}{2^{13}})n + (\frac{1}{2^3} + \frac{1}{2^4}), n > 70 \wedge n \leq 256. \end{cases}$$

Fig. 6.38 shows the plots of $connect_appr(n)$ and $connect(n)$ depending on the values $n \in [4, 256]$.

Therefore, a high-precision approximation of the original formula of cubic functions connection point is ensured.

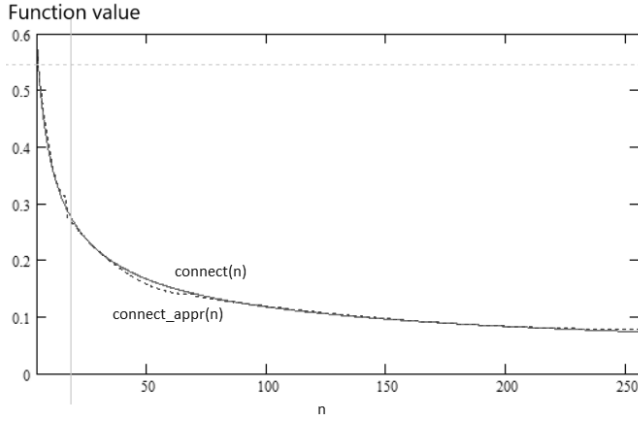


Fig. 6.38. The plots of original and approximate formulas of calculation of the cubic functions connection point

To ensure the smoothness of the connection of two cubic functions, a 0.5 level was chosen. Finding the derivatives of functions F_{KUB} and F_{KUB22} is not necessary, since the values of functions are similar at a sufficiently large interval near $connect(n)$ (Fig. 6.39 shows the absolute deviation between F_{KUB} and F_{KUB22} for $n \in [50, 200]$).

Fig. 6.40 shows the plots of F_{KUB} , $F_{KUBCOMB}$ and F_B when $n = 50$.

Through using the combination of two cubic functions in $F_{KUBCOMB}$, an increase in the accuracy of the approximation of the glare's attenuation zone has been achieved.

Since the F_{KUB} and $F_{KUBCOMB}$ coincide in the zone of glare's epicenter, their maximum relative errors δ of F_B approximation in the epicenter are equal (3%, Fig. 6.41).

Fig. 6.42 shows the plots of maximum absolute errors Δ of F_{KUB} and $F_{KUBCOMB}$ from F_B depending on $n \in [4, 256]$.

It is worth noting that the maximum absolute error of F_{KUB} from the known function is 0.11, it's unacceptable. Maximum absolute error of $F_{KUBCOMB}$ from F_B is 0.035. Therefore, $F_{KUBCOMB}$ in comparison with F_{KUB} provides more accurate F_B approximation.

Fig. 6.43 shows the plot of absolute error between $F_{KUBCOMB}$ and F_B depending on $n \in [4, 256]$ and $x \in [0, \pi/2]$ values.

Based on F_{KUB} and $F_{KUBCOMB}$ the test figure "Teapot" was visualized in BRDF Explorer. The visualization result is given in Fig. 6.44.

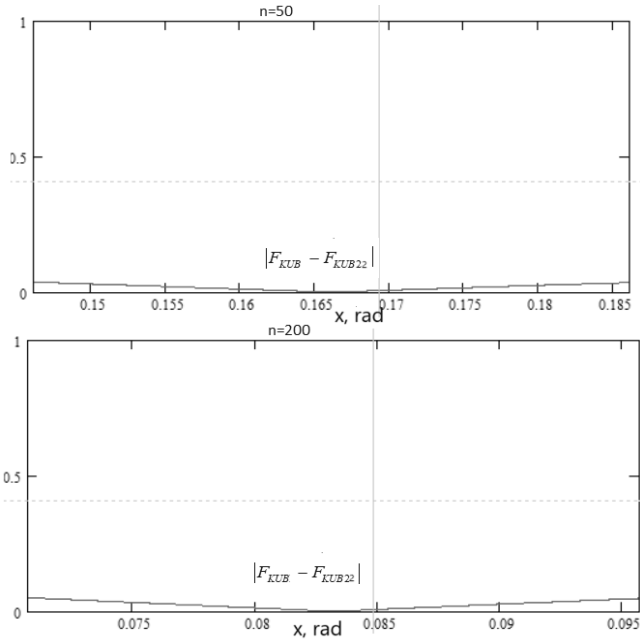


Fig. 6.39. The plots of absolute deviation between F_{KUB} and F_{KUB22} near their connection point

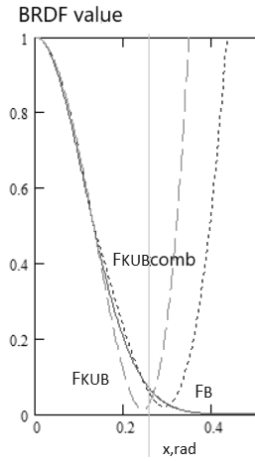


Fig. 6.40. The plots of F_{KUB} , $F_{KUBcomb}$ and F_B when $n = 50$

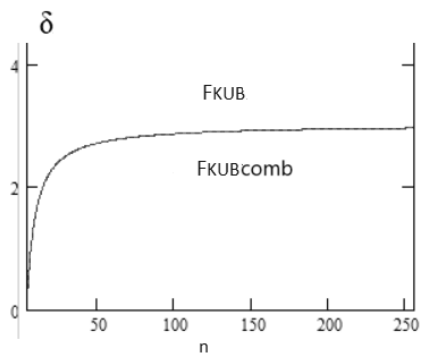


Fig. 6.41. The plots of maximum relative errors of F_{KUB} , $F_{KUBCOMB}$ in glare's epicenter

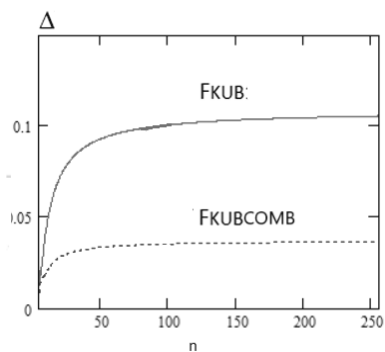


Fig. 6.42. The plots of maximum absolute errors of F_{KUB} , $F_{KUBCOMB}$ from F_B

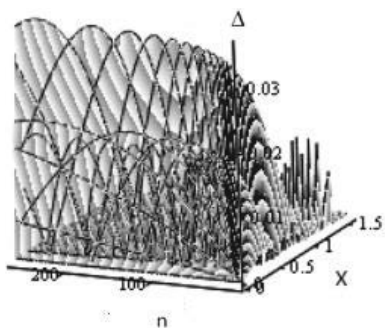


Fig. 6.43. The plot of absolute errors between $F_{KUBCOMB}$ and F_B depending on angle value and shininess



Fig. 6.44. The visualized teapots based on F_{KUB} , $F_{KUBCOMB}$

Fig. 6.44 shows that $F_{KUBCOMB}$ provides a more realistic formation of the glare's attenuation zone.

To ensure highly realistic shading of surfaces of three-dimensional objects, BRDF must meet the condition of compliance with the law of conservation of energy. Therefore, we will consider the development of energy-correct BRDFs.

The condition for complying with the law of conservation of energy can be expressed through the integral expression [23]

$$\int_{\Omega} f_r(\vec{L}, \vec{V}) * \cos(x) d\omega \leq 1,$$

where $d\omega$ is the solid angle, calculated as $\sin(x)dx d\varphi$, $f_r(\vec{L}, \vec{V})$ — BRDF.

Let's calculate the normalizing coefficient for the improved Schlick model F_{MSH} , with which the model will meet the specified condition.

We denote the desired coefficient as $coef(n)$ and form the equation for finding the solution. We take into account the condition of maximization of the reflected energy

$$coef(n) \int_{\Omega} f_r(\vec{L}, \vec{V}) * \cos(x) d\omega = 1.$$

We substitute F_{MSH} into the expression instead of $f_r(\vec{L}, \vec{V})$, the angle ψ between $\vec{R}, \vec{V}$ instead of x [23]

$$coef(n) * \int_{\Omega} \frac{2 \cos(\psi)}{1.25(n - n * \cos(\psi) + 1.25 \cos(\psi))^2} \cos(\psi) d\omega = 1.$$

We move to the spherical coordinate system. The value at which F_{MSH} reaches zero is approximately $\frac{\pi}{2}$. Then we obtain the expression of integral

$$\int_0^{2\pi} \int_0^{\pi/2} \frac{2\cos(\psi)}{1.25(n - n^* \cos(\psi) + 1.25\cos(\psi))^2} \cos(\psi) \sin(\psi) d\psi d\varphi .$$

Let us find the value of the definite integral with respect to $d\varphi$. The integral takes the form

$$2\pi \int_0^{\pi/2} \frac{2\cos(\psi)}{1.25(n - n^* \cos(\psi) + 1.25\cos(\psi))^2} \cos(\psi) \sin(\psi) d\psi .$$

For the expression of the integral, we introduce the designation $Int(n)$

$$Int(n) = \int_0^{\pi/2} \frac{2\cos(\psi)}{1.25(n - n^* \cos(\psi) + 1.25\cos(\psi))^2} \cos(\psi) \sin(\psi) d\psi .$$

Then the equation for finding the normalizing coefficient has the form $coef(n) * 2\pi * Int(n) = 1$.

From the equation we determine the normalizing coefficient $coef(n)$ for particular n using the formula

$$coef(n) = \frac{1}{2\pi * Int(n)} .$$

We calculate the value $coef(n)$ for each n in the interval $n \in [1, 1000]$. Next, we save the sample of $coef(n)$ values of size 50 in a text file (Fig. 6.45). Based on the calculated data, we determine the formula of the $coef(n)$ value dependence on n .

n , $coef(n)$
1, 0.419
2, 0.6
3, 0.765
5, 1.073
7, 1.367
9, 1.653
10, 1.793
30, 4.476
40, 5.779
50, 7.071
70, 9.634

Fig. 6.45. Part of the calculated ideal coefficient values for n

The symbolic regression software TuringBot was used to find the formula of the normalizing coefficient of F_{MSH} . A text file with a data sample

was used as input data for the calculation, and the root mean square error (RMS) was chosen as the metric (Fig. 6.46).

Input		Advanced
Input file	3нач5.txt	
Target variable	x	
Input variables	n ☒ Row number ☐	
All/none		
Search options		
Search metric	RMS error	
Train/test split	No cross-validation	
Test sample	Chosen randomly	

Fig. 6.46. Selecting settings for calculating the formula of the normalizing coefficient of the modified Schlick model in TuringBot

For the input data, the program obtained a graph of the dependence of the normalizing coefficient ideal values from n (Fig. 6.47, the abscissa axis is the number in the sample, the ordinate axis is the value of the coefficient).

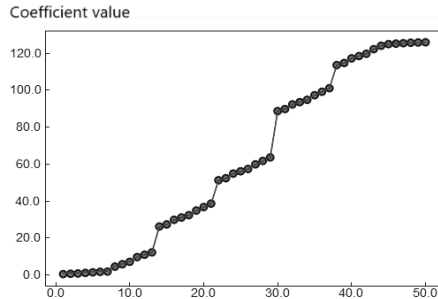


Fig. 6.47. Graph of ideal $coef(n)$ values relative to growth of n

As a result, a set of variants of normalizing coefficients formulas of different levels of accuracy and complexity was obtained (Fig. 6.48).

The last formula from the list was chosen, taking into account the accuracy and acceptable complexity of the calculation. Therefore, the resulting formula for the normalizing coefficient of the modified Schlick model is calculated as [23]:

$$0.125n - \left(\frac{45.76}{n + 46.578} \right) + 1.317.$$

Solutions

Size	Error	Function
1	46,102308	62.2421
3	0,504588	0.126348*n
5	0,188049	0.12533*(6.22623+n)
8	0,111746	0.122027*(sqrt(n)+n)
10	0,063278	(0.124905-(1.12537/(-9.64838-n)))*n
12	0,020065	0.124723*n-(45.76/(n+46.5778))+1.31686

Fig. 6.48. Obtained variants of the normalizing coefficient formulas for the modified Schlick model

Figure 6.49 shows a plot of the absolute error (Δ_{HEMI}) of hemispherical integral reflectance ($coef(n) * 2\pi * Int(n)$) from 1. The maximum error in the interval $n \in [2, 1000]$ does not exceed $42 * 10^{-3}$.

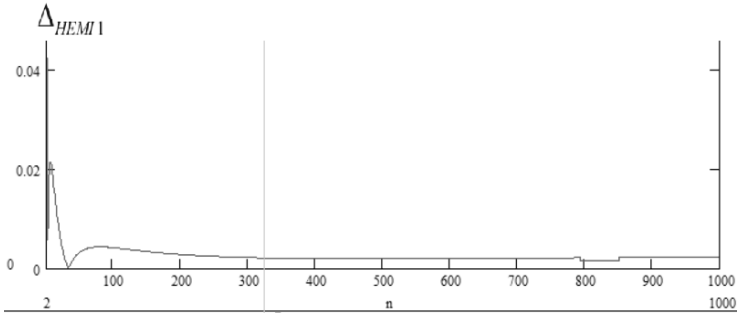


Fig. 6.49. Deviation of hemispherical integral reflectance from 1

Therefore, the developed model is energetically-correct and complies with the law of conservation of energy.

Let us calculate the normalizing coefficient for the BRDF based on calculating the angle between vectors of the form [14]

$$\cos\left(\sqrt{\frac{n}{k}} \cdot x\right)^k,$$

where k is the coefficient, usually $k \in [1, 5]$.

We denote this function as F_{ang} . First, we select a coefficient for the root expression of F_{ang} in order to increase the realism of the reproduction of the epicenter of the glare. We denote the new model as $F_{mod ang}$. The model is calculated using the formula

$$\cos\left(\sqrt{\frac{a \cdot n}{k}} \cdot x\right)^k,$$

where a — the desired coefficient.

Possible values of the coefficient a belong to the interval $[0.05, 2]$. The step between the values of the interval is set to 0.05 . A computer program has been developed to determine the value of the coefficient. In the loop, for each possible a value the function of deviation (*ErrorWithBlinn*) between $F_{\text{mod ang}}$ and F_B in the epicenter zone is calculated. The a value is chosen, which provides the smallest total value of the relative deviation function. The flowchart of the calculation of the deviation function *ErrorWithBlinn* is shown in Fig. 6.50.

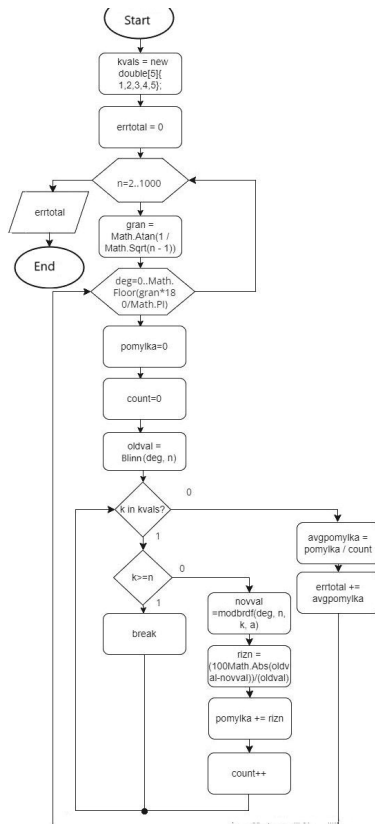


Fig. 6.50. Flowchart of the function *ErrorWithBlinn* between $F_{\text{mod ang}}$ and F_B

Array of possible values of k *kvals* is initialized with integers in the range $[1,5]$, since it is advisable to choose the smallest k values instead of relatively large ones n . A variable *errtotal* is used to store the final error value.

In a double cycle, the F_B value (variable *oldval*) is determined with respect to the angle $\deg$ and n . For each n , the value of the limit point (variable *gran*) of the glare's epicenter zone calculation is determined.

For each element of the array $k \in [1,5]$ the $F_{\text{mod ang}}$ values (variable *novval*) are determined. If k reaches the n value, the iteration through the k array ends. The *rizn* variable stores the relative errors between $F_{\text{mod ang}}$ (for different k) and F_B . The *pomylka* variable is the sum of *rizn*, *avgpomylka* — the average value of *rizn*. The average values of the relative error are added to the final error value *errtotal*.

As a result of the program, the optimal a value is determined, which is equal to 0.95. Then $F_{\text{mod ang}}$ is calculated using the formula [14]

$$\cos\left(\sqrt{\frac{0.95n}{k}}x\right)^k.$$

Fig. 6.51 shows the plots of $F_{\text{mod ang}}$, F_{ang} , F_{78} ($a = \frac{7}{8}$), F_B in the glare's epicenter zone when $n = 300, k = 3$.

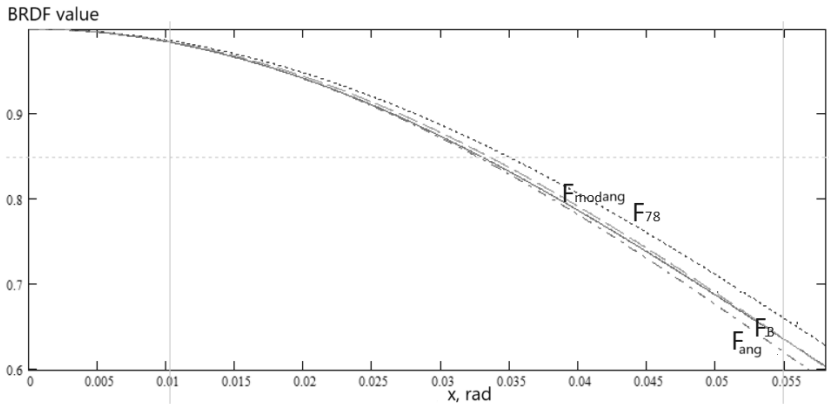


Fig. 6.51. Plots of $F_{\text{mod ang}}$, F_{ang} , F_{78} , F_B in the epicenter zone when $n = 300, k = 3$

Fig. 6.52 shows plots of maximum relative errors (δ) in the epicenter zone between $F_{\text{mod ang}}$, F_{ang} , F_{78} and F_B relative to $n \in [6,1000], k \in [1,5]$.

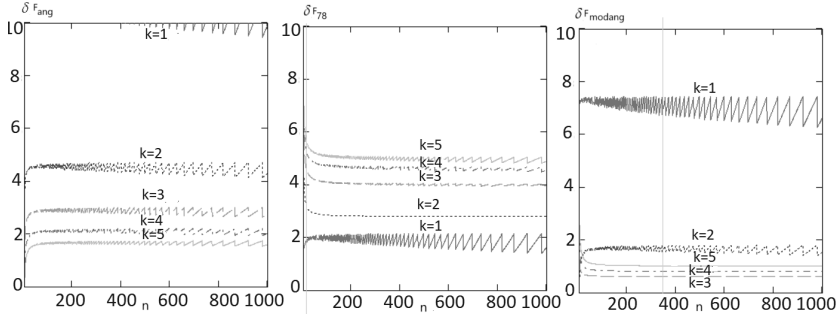


Fig. 6.52. Maximum δ of F_{ang} , F_{78} , F_{modang} from F_B relative to $n \in [6, 1000], k \in [1, 5]$

Therefore, for F_{ang} the smallest maximum δ relative to $n \in [6, 1000]$, $k \in [1, 5]$ is 1.7 % ($k = 5$), for F_{modang} — 0.8 % ($k = 3$).

F_{modang} provides more accurate formation of glare in the epicenter area than F_{ang} .

Next, we find the normalizing coefficient for F_{modang} . We write the equation in a spherical coordinate system [14]

$$coef(n, k) \int_0^{2\pi} \int_0^{gran(n, k)} \cos\left(\sqrt{\frac{0.95n}{k}}\psi\right)^k \cos(\psi) \sin(\psi) d\psi d\varphi = 1,$$

$$\text{where } gran(n, k) = \frac{1.612}{\left(\frac{n}{k}\right)^{\frac{1}{2}}}.$$

We integrate the left part of the equation with respect to $d\varphi$. We obtain the expression

$$coef(n, k) \cdot 2\pi \cdot \int_0^{gran(n, k)} \cos\left(\sqrt{\frac{0.95n}{k}}\psi\right)^k \cos(\psi) \sin(\psi) d\psi = 1.$$

We denote the integral of the left part of the equation as $Int(n, k)$

$$Int(n, k) = \int_0^{gran(n, k)} \cos\left(\sqrt{\frac{0.95n}{k}}\psi\right)^k \cos(\psi) \sin(\psi) d\psi.$$

We obtain an equation for finding the normalizing coefficient for the modified model based on calculating the angle between the vectors.

$$coef(n, k) \cdot 2\pi \cdot Int(n, k) = 1.$$

We calculate the value $coef(n,k)$ relatively to n, k ($n \in [2,1000], k \in [1, n-1]$)

$$coef(n,k) = \frac{1}{2\pi * Int(n,k)} .$$

The selected 50 n, k , $coef(n,k)$ values are saved in a text file (Fig. 6.53).

```
n,k,coef(n,k)|
2,1,0.701
3,1,0.957
3,2,0.809
5,1,1.481
9,2,2.029
5,3,1.129
5,4,1.092
20,1,5.447
25,2,5.321
29,3,5.605
21,4,3.921
```

Fig. 6.53. Sample of $n, k, coef(n,k)$ values

The text file values were used to select the $coef(n,k)$ formula in the TuringBot software. Error metric — RMS error (Fig. 6.54).

The screenshot shows the TuringBot software interface. The 'Input file' field is set to 'norm.txt'. The 'Target variable' is 'y'. The 'Input variables' are 'n' and 'k', both with checkboxes checked. The 'Row number' checkbox is unchecked. The 'Search options' section shows 'Search metric' as 'RMS error', 'Train/test split' as 'No cross-validation', and 'Test sample' as 'Chosen randomly'.

Fig. 6.54. Setting input data and error metrics for $coef(n,k)$ formula selection in TuringBot

A set of $coef(n,k)$ possible formulas is obtained (Fig. 6.55). The last expression from the list is selected, taking into account the smallest value of the error metric.

So, the $coef(n,k)$ calculation formula of F_{modang} is [14]

$$coef(n,k) = \left(\frac{0.11}{k-0.07} + 0.15 \right) \cdot (n-0.48) + 0.28 .$$

Fig. 6.56 shows the maximum absolute deviations Δ_{HEMI2} of the hemispherical integral reflectivity from 1 for $k=1,5$ at intervals $n \in [k+1,1000]$.

Size	Error	Function
1	68,393486	63.9013
3	19,278193	$0.190961 \cdot n$
5	18,358105	$0.198771 \cdot (n-k)$
7	16,155554	$(0.204499 + (-0.000496315 \cdot k)) \cdot n$
8	0,429636	$((0.113777/k) + 0.150481) \cdot n$
10	0,143631	$((0.105896/(-0.0694905+k)) + 0.151319) \cdot n$
12	0,061396	$((0.105837/(-0.0699320+k)) + 0.151066) \cdot n + 0.180739$
14	0,059692	$((0.105868/(-0.0702525+k)) + 0.151038) \cdot (n-0.48627) + 0.278514$

Fig. 6.55. Variants of $coef(n, k)$ calculation formulas in TuringBot

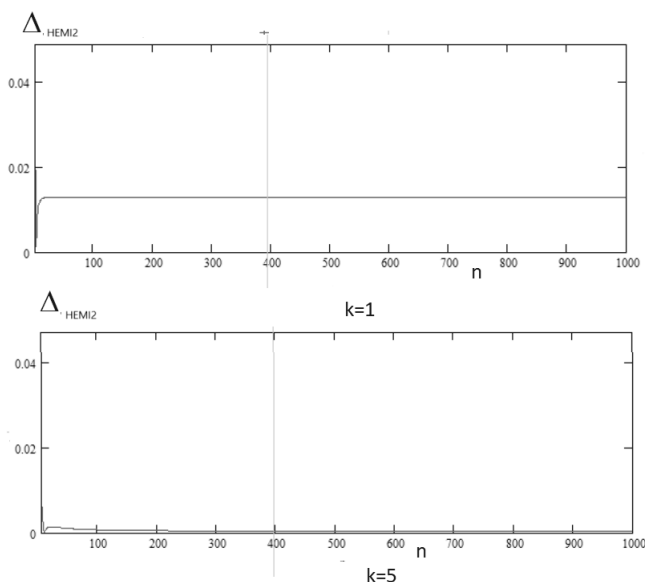


Fig. 6.56. Dependence of Δ_{HEMI2} from n for $k=1,5$

The maximum Δ_{HEMI2} value for $k \in [1, 999]$ on the intervals $n \in [k+1, 1000]$ is $2 \cdot 10^{-2}$.

Let's calculate the normalizing coefficient for another form of the reflectance model based on calculating the angle between the vectors ($F_{kur \log}$) [15]

$$\cos^k \left(\frac{\frac{\ln Q}{k} \cdot \arccos(e^{\frac{\ln Q}{k}})}{\arccos(\text{signum}(Q)^n \cdot |Q|^{\frac{1}{n}})} \cdot x \right).$$

Given that $Q > 0$, we simplify the expression

$$\cos^k \left(\frac{a \cos(e^{\frac{\ln Q}{k}})}{a \cos(Q^{\frac{1}{n}})} \cdot x \right).$$

Taking into account the condition of maximizing the energy of reflected light and switching to a spherical coordinate system, we obtain the equation

$$coef(n, k, Q) \int_0^{2\pi} \int_0^{gran(n, k, Q)} \cos^k \left(\frac{a \cos(e^{\frac{\ln Q}{k}})}{a \cos(Q^{\frac{1}{n}})} \cdot \psi \right) \cos(\psi) \sin(\psi) d\psi d\varphi = 1,$$

where $coef(n, k, Q)$ is the desired normalizing coefficient,

$$gran(n, k, Q) = \frac{\pi \cdot a \cos(Q^{\frac{1}{n}})}{2 \cdot a \cos(Q^{\frac{1}{k}})}.$$

We integrate the left side of the equation with respect to $d\varphi$. We obtain

$$coef(n, k, Q) \cdot 2\pi \cdot \int_0^{gran(n, k, Q)} \cos^k \left(\frac{a \cos(e^{\frac{\ln Q}{k}})}{a \cos(Q^{\frac{1}{n}})} \cdot \psi \right) \cos(\psi) \sin(\psi) d\psi = 1.$$

We denote the integral expression as $Int(n, k, Q)$

$$Int(n, k, Q) = \int_0^{gran(n, k, Q)} \cos^k \left(\frac{a \cos(e^{\frac{\ln Q}{k}})}{a \cos(Q^{\frac{1}{n}})} \cdot \psi \right) \cos(\psi) \sin(\psi) d\psi.$$

According to the equation, we calculate the value of $coef(n, k, Q)$. The value of $coef(n, k, Q)$ is calculated for $n \in [1, 2, 3, 4, 5, 7, 10, 20, 50, 100, 200, 500, 1000]$, $k \in [1; 5]$, $Q \in [0.1; 0.9]$.

Based on the obtained sets of n , k , Q , $coef(n, k, Q)$ values, two files were formed for finding formulas of $coef(n, k, Q)$ for $Q \in [0.1; 0.4]$ and $Q \in [0.5; 0.9]$ respectively (Fig. 6.57)

To find the $coef(n, k, Q)$ formulas, the SymbolicRegressor module of the Python library gplearn was used. The module combines symbolic regression and genetic algorithms.

We create a set of possible operations of the desired formula function _set (Fig. 6.58), which includes addition ('add'), subtraction ('sub'), multiplication ('mul'), and division ('div').

p, k, n, y	Q, k, n, y
0.5, 1, 1, 0.477	0.1, 1, 1, 0.477
0.5, 1, 2, 0.699	0.1, 1, 2, 0.569
0.5, 1, 3, 0.88	0.1, 1, 5, 0.928
0.5, 1, 4, 1.095	0.1, 1, 10, 1.57
0.5, 1, 5, 1.313	0.1, 1, 20, 2.873
0.5, 1, 7, 1.75	0.1, 1, 50, 6.797
0.5, 1, 10, 2.41	0.1, 1, 100, 13.344
0.5, 1, 20, 4.612	0.1, 1, 200, 26.438
0.5, 1, 50, 11.227	0.1, 1, 500, 65.722
0.5, 1, 100, 22.255	0.1, 1, 1000, 131.196
0.5, 1, 200, 44.312	
0.5, 1, 500, 110.482	
0.5, 1, 1000, 220.765	

Fig. 6.57. Generated files with values $n, k, Q, coef(n, k, Q)$

```
'function_set' = ['add', 'sub', 'mul', 'div']
```

Fig. 6.58. Formation of a set of possible operations of $coef(n, k, Q)$

We set up the $coef(n, k, Q)$ formula selection procedure (Fig. 6.59). We experimentally set the population size to 300 for the first $coef(n, k, Q)$ formula selection, 400 for the second formula selection. We set the number of generations to 1000.

```
st_gp = SymbolicRegressor(population_size=300, function_set=function_set,
                          generations=1000, stopping_criteria=0.01,
                          p_crossover=0.7, p_subtree_mutation=0.1,
                          p_hoist_mutation=0.05, p_point_mutation=0.1,
                          max_samples=0.9, verbose=1,
                          parsimony_coefficient=0.01, random_state=0, feature_names=X_train.columns)

st_gp = SymbolicRegressor(population_size=400, function_set=function_set,
                          generations=1000, stopping_criteria=0.01,
                          p_crossover=0.7, p_subtree_mutation=0.1,
                          p_hoist_mutation=0.05, p_point_mutation=0.1,
                          max_samples=0.9, verbose=1,
                          parsimony_coefficient=0.01, random_state=0, feature_names=X_train.columns)

st_gp.fit(X_train, y_train)
```

Fig. 6.59. Setting up the procedure for selecting normalizing coefficient formulas

Respective to the $Q \in [0.1; 0.4]$ values, we obtain the formula for the normalizing coefficient [15]

$$\frac{0.608 \cdot n \cdot (Q - 0.193)}{2Q + 2k} + 0.161n + 0.284.$$

The accuracy of the approximation of the dataset (coefficient of determination R2) is 0.99977 (Fig. 6.60).

998	30.94	456590	31	0.181371	0.647548	0.65s
999	31.37	1109.82	31	0.193823	0.537968	0.00s
R2: 0.9997662040448603						
out[27]: $\frac{0.608n(Q-0.193)}{2Q+2k} + 0.161445672n + 0.283821491376$						

Fig. 6.60. Result of selecting the normalizing coefficient formula for the range of values $Q \in [0.1; 0.4]$

The obtained formula for the range $Q \in [0.5; 0.9]$ is

$$\frac{0.743Q \cdot (-0.0133kn + 0.167n)}{k} + 0.167n + 0.245.$$

The accuracy of the approximation of the dataset is 0.99999 (Fig. 6.61).

998	36.05	588.93	37	0.0832429	0.238803	0.98s
999	36.83	247.427	37	0.083084	0.240233	0.00s
R2: 0.9999884733152802						
$\frac{0.743044Q(-0.013319008848kn + 0.167n)}{k} + 0.167n + 0.245$						

Fig. 6.61. Result of selecting the normalizing coefficient formula for the range of values $Q \in [0.5; 0.9]$

Therefore, the formula for the normalizing coefficient for $F_{kut \log}$ is calculated as [15]

$$\begin{cases} \frac{0.608 \cdot n \cdot (Q - 0.193)}{2Q + 2k} + 0.161n + 0.284, Q \in [0.1; 0.4] \\ \frac{0.743Q \cdot (-0.0133kn + 0.167n)}{k} + 0.167n + 0.245, Q \in [0.5; 0.9]. \end{cases}$$

Table 6.4 provides information on the absolute errors Δ_{HEMI3} of the hemispherical integral reflectivity from 1 relative to $k \in [1; 5]$, $Q \in [0.1; 0.9]$ at intervals $n \in [k + 1; 1000]$.

Fig. 6.62 shows a plot of Δ_{HEMI3} values for the interval $n \in [6; 1000]$ when $k = 5, Q = 0.5$.

Therefore, the maximum Δ value for $k \in [1; 5]$, $Q \in [0.1; 0.9]$, $n \in [k + 1; 1000]$ is $91 \cdot 10^{-3}$.

Table 6.4

Values of maximum Δ_{HEMI3} depending on Q, k

Q / k	1	2	3	4	5
0.1	$42 \cdot 10^{-3}$	$49 \cdot 10^{-3}$	$49 \cdot 10^{-3}$	$43 \cdot 10^{-3}$	$38 \cdot 10^{-3}$
0.2	$31 \cdot 10^{-3}$	$21 \cdot 10^{-3}$	$21 \cdot 10^{-3}$	$21 \cdot 10^{-3}$	$15 \cdot 10^{-3}$
0.3	$47 \cdot 10^{-3}$	$17 \cdot 10^{-3}$	$10 \cdot 10^{-3}$	$10 \cdot 10^{-3}$	$10 \cdot 10^{-3}$
0.4	$67 \cdot 10^{-3}$	$47 \cdot 10^{-3}$	$38 \cdot 10^{-3}$	$35 \cdot 10^{-3}$	$30 \cdot 10^{-3}$
0.5	$42 \cdot 10^{-3}$	$16 \cdot 10^{-3}$	$11 \cdot 10^{-3}$	$11 \cdot 10^{-3}$	$11 \cdot 10^{-3}$
0.6	$46 \cdot 10^{-3}$	$23 \cdot 10^{-3}$	$17 \cdot 10^{-3}$	$13 \cdot 10^{-3}$	$13 \cdot 10^{-3}$
0.7	$65 \cdot 10^{-3}$	$35 \cdot 10^{-3}$	$24 \cdot 10^{-3}$	$21 \cdot 10^{-3}$	$16 \cdot 10^{-3}$
0.8	$78 \cdot 10^{-3}$	$46 \cdot 10^{-3}$	$35 \cdot 10^{-3}$	$29 \cdot 10^{-3}$	$23 \cdot 10^{-3}$
0.9	$91 \cdot 10^{-3}$	$60 \cdot 10^{-3}$	$44 \cdot 10^{-3}$	$33 \cdot 10^{-3}$	$28 \cdot 10^{-3}$

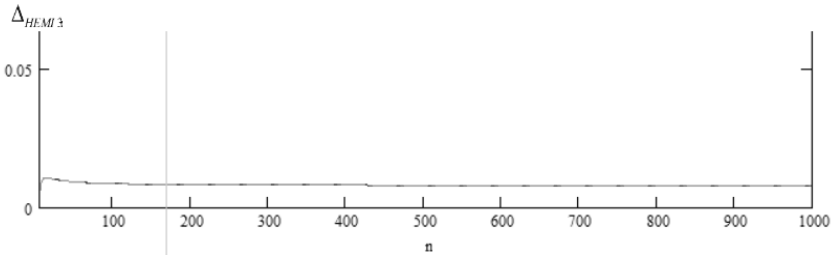


Fig. 6.62. Plot of Δ_{HEMI3} values for the interval $n \in [6, 1000]$ when $k = 5, Q = 0.5$

An example of forming a three-dimensional object using the obtained BRDF in the Idx3d tool is shown in Fig. 6.63.



Fig. 6.63. Example of shading a three-dimensional graphic figure using the energetically-correct $F_{kut \log}$

Let's calculate the normalizing coefficient for the logarithmic BRDF F_{log} [16].

We substitute the formula of F_{log} into the equation of BRDF energetical correctness, next we move to the spherical coordinates. The equation takes the form [16]

$$\text{coef}(n) \int_0^{2\pi} \int_0^{\text{gran}(n)} (1 - 0.5 \cdot \log_2(1 - n \cdot \log_2(\cos(\psi)))) \cdot \cos(\psi) \sin(\psi) d\psi d\varphi = 1,$$

where $\text{gran}(n) = \text{acos}(e^{-\frac{2.08}{n}})$.

After the integration of the left part by $d\varphi$ the equation takes the form

$$\text{coef}(n) \cdot 2\pi \cdot \int_0^{\text{gran}(n)} (1 - 0.5 \cdot \log_2(1 - n \cdot \log_2(\cos(\psi)))) \cdot \cos(\psi) \sin(\psi) d\psi = 1.$$

The integral expression is replaced by the $\text{Int}(n)$

$$\text{Int}(n) = \int_0^{\text{gran}(n)} (1 - 0.5 \cdot \log_2(1 - n \cdot \log_2(\cos(\psi)))) \cos(\psi) \sin(\psi) d\psi.$$

Then the normalizing coefficient $\text{coef}(n)$ is calculated using the formula

$$\text{coef}(n) = \frac{1}{2\pi \cdot \text{Int}(n)}.$$

The values of $\text{coef}(n)$ were calculated depending on $n \in [1, 1000]$. The sample of 50 $n, \text{coef}(n)$ pairs was saved in the text file (Fig. 6.64).

$n, \text{coef}(n)$
1, 0.483
2, 0.665
3, 0.855
4, 1.048
5, 1.243
6, 1.438
7, 1.634

Fig. 6.64. The saved in the text file sample of $n, \text{coef}(n)$ pairs

The data of the text file were used for the $\text{coef}(n)$ formula selection using the software tool TuringBot (Fig. 6.65).

The root mean square error (RMS) was chosen as the error metric. Among the possible operations of the formula the addition, multiplication and division were chosen (Fig. 6.66).

Fig. 6.65. Usage of the text file for $\text{coef}(n)$ formula selection in TuringBot

Fig. 6.66. The setting of error metric and possible formula operations in TuringBot

As a result, the set of possible $\text{coef}(n)$ formulas was received (Fig. 6.67). The second last formula was chosen from the list, taking into account its simplicity and low error value.

Size	Error	Function
1	62.583411	58.7142
3	0.183519	$0.197639 \cdot n$
5	0.006839	$0.197244 \cdot (n + 1.27235)$
9	0.006503	$(0.197234 - (-1.39266e-08 \cdot n)) \cdot (n + 1.2752)$
10	0.001128	$0.197253 \cdot ((0.224161 \cdot n) + 1.2449 + n)$
12	0.000280	$0.197254 \cdot (n + 1.23891) - (0.0681339 / (0.641677 + n))$
18	0.000273	$0.197254 \cdot (n - (0.242046 / ((-0.680778 + (-0.00838297 \cdot n)) \cdot n - 0.475525))) + 1.24108$

Fig. 6.67. Set of possible $\text{coef}(n)$ formulas in TuringBot

Therefore, $\text{coef}(n)$ for $F_{\log}$ is calculated using the formula [16]

$$0.197(n + 1.24) + \frac{0.068}{0.64 + n}.$$

Fig. 6.68 shows the plot of absolute errors Δ_{HEMI4} between hemispherical integral reflectance and 1 depending on $n \in [1, 1000]$.

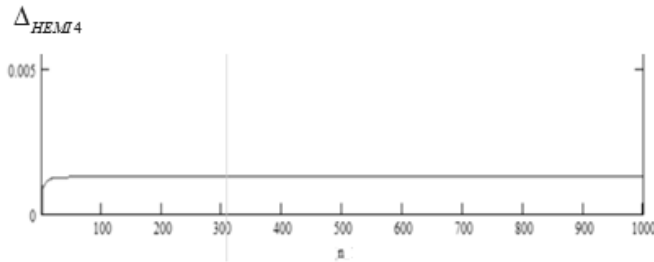


Fig. 6.68. Plot of Δ_{HEMI4} depending on $n \in [1, 1000]$

The maximum Δ_{HEMI4} value at the interval $n \in [1, 1000]$ is $1 \cdot 10^{-3}$.

For the visualization of the use of the modified logarithm function based reflectance model the web tool OneShader was used.

Fig. 6.69 shows the code of F_{log} implementation using GLSL (Graphics Library Shading Language) in OneShader. Fig. 6.70 shows the visualized spheres using the energetically correct logarithm function based model, original logarithm function based model, Schlick model.

```
float speclog=min(max((0.197*(n+1.24)+0.068/(0.64+n))*
(1.0-0.5*log2(1.0-n*log2(dot(N, H))))),0.0),1.0);
```

Fig. 6.69. Code of energetically correct F_{log} implementation using GLSL

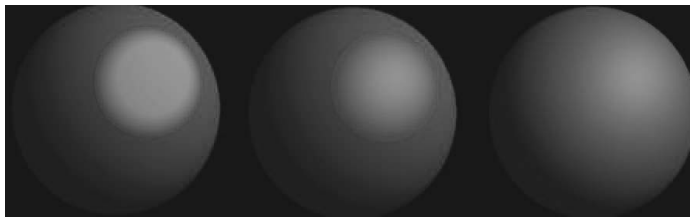


Fig. 6.70. Created spheres images based on energetically-correct F_{log} , original F_{log} , F_{SH} respectively

Summarizing, the usage of energetically correct logarithm function based model allows more precise glow's epicenter and attenuation zones reproduction in comparison with F_{log} .

6.4. ADAPTIVE SELECTION OF BIDIRECTIONAL REFLECTANCE DISTRIBUTION FUNCTIONS

Each of the available BRDF has its own application features, depending on the characteristics of the object's surface material and the computing capabilities of the graphics system.

Therefore, let us consider the proposed method of adaptive selection of BRDF.

Adaptive selection of the bidirectional reflectance distribution function for the material is proposed to be carried out according to the flowchart in Fig. 6.71 [24].

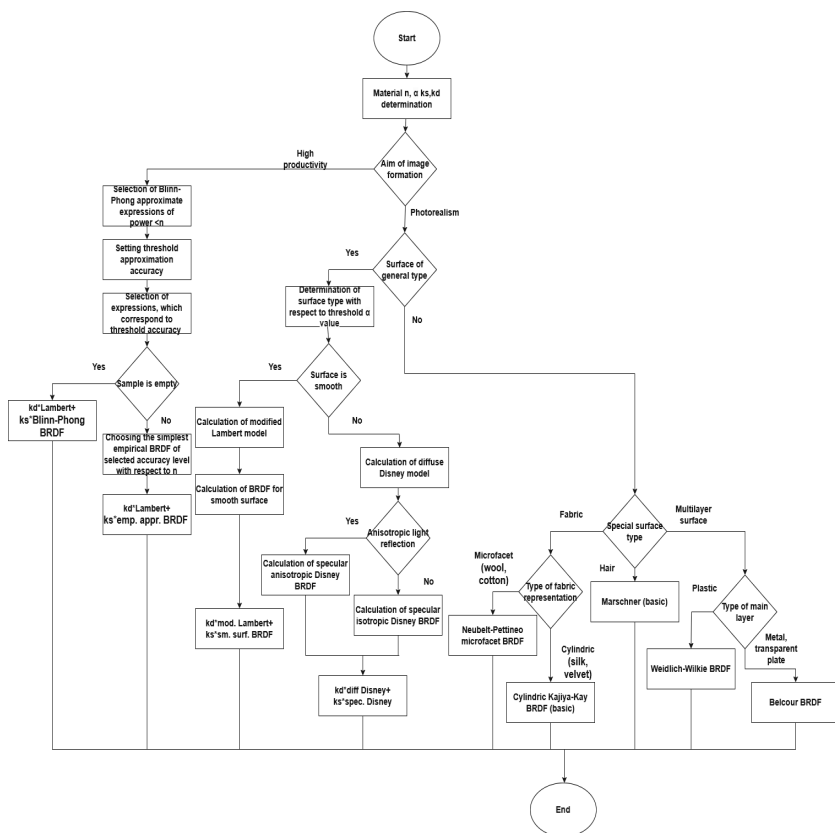


Fig. 6.71. Flowchart of adaptive selection of BRDF for material

First n , α , k_d , k_s values are determined from special tables for the material.

After obtaining tabular characteristics of the material, the main goal of image formation is determined — high performance or photorealism.

High-performance shading is used when there are significant time and hardware constraints on rendering a scene (for example, rendering frames in interactive mobile games).

If sufficient computational resources are available to take into account the microstructure and complex optical properties of the object's surface, photorealistic images are formed (for example, when developing cartoon frames or visualizing game frames on a powerful gaming computer).

In the case of high-performance scene visualization, it is proposed to use the simplest diffuse and specular lighting models — Lambert (formula (6.1)) and Blinn–Phong (formula (6.6)) respectively.

In this case, the Blinn–Phong model is approximated by a simpler one, if possible. To do this, the presence of approximation expressions with a degree less than n is checked. As approximation the expressions F_2 , F_4 , F_8 , F_{16} , F_{SH} , F_{KV} , F_{KUB} can be considered.

The threshold level of relative (Table 6.5) [24] and absolute (Table 6.6) error [24] of approximation of the original model is selected.

Among the expressions that correspond to the selected level of approximation error, the formula that is most easily calculated is selected. It is proposed to determine the simplicity of the formula based on the number of clock cycles of the AMD Zen 4 processor (2022) [24], listed in Table 6.7. The table also shows the additional hardware costs for calculating simplified models.

Table 6.5

Maximum relative errors (δ , %) of Blinn-Phong approximation models from the original model in the glare epicenter zone

Model/ n	2	5	10	20	50	100	200	500	700	1000
F_2	0	5	6.5	7	7	7	7	7	7	7
F_4	6	1	2	3	3	3	3	3	3	3
F_8	9	1	0.5	1	1.5	1.5	1.5	1.5	1.5	1.5
F_{16}	10	2	0.5	0.25	0.5	0.5	0.75	0.75	0.75	0.75
F_{SH}	9	10	10	10	10	10	10	10	10	10
F_{KV}	23	8	4	2	3	3	3	3	3	3
F_{KUB}	0	1	2	2	2.5	3	3	3	3	3

Table 6.6

**Maximum absolute errors of Blinn-Phong approximation models
from the original model**

Model /n	2	5	10	20	50	100	200	500	700	1000
F_2	0	0.11	0.14	0.15	0.16	0.16	0.16	0.16	0.16	0.16
F_4	0.09	0.02	0.05	0.06	0.07	0.07	0.07	0.07	0.07	0.07
F_8	0.13	0.02	0.01	0.02	0.03	0.03	0.03	0.03	0.03	0.03
F_{16}	0.15	0.04	0.01	0.003	0.01	0.02	0.02	0.02	0.02	0.02
F_{SH}	0.099	0.16	0.18	0.19	0.2	0.2	0.2	0.2	0.2	0.2
F_{KV}	0.15	0.23	0.25	0.27	0.27	0.27	0.27	0.27	0.27	0.27
F_{KUB}	0	0.02	0.05	0.07	0.09	0.1	0.1	0.1	0.11	0.11

Table 6.7

**Number of clock cycles of AMD Zen 4 processor operations
for calculating Blinn-Phong approximation models**

Model	ADD latency (clock cycles)	N_{op} (numeric operations)	MUL latency	N_{op}	DIV latency	N_{op}	Total latency	Additional re- sources
F_2	1	2	3	2	10	0	8	Saving 1000 coordinates of the point after which zeroing is required or +10 latency on its calculation
F_4		2		3		0	11	
F_8		2		4		0	14	
F_{16}		2		5		0	17	
F_{KUB}		2		5		0	17	
F_{KV}		2		4		0	14	Saving 1000 coordinates of the point, after which the truncation of negative values is required or +1 latency for deter- mining the nega- tivity of a function value
F_{SH}		2		1		1	15	—

If the approximation expressions are characterized by an approximation error that exceeds the threshold, the Blinn-Phong model is used. The selected specular model and the Lambert model are combined to calculate the pixel color intensity, taking into account the defined for the material k_d, k_s values.

If the computational capabilities of the rendering system allow the generation of photorealistic images based on complex reflectance models, the proposed scheme provides various options for calculating the BRDF for general-type surfaces, as well as for hair, fabrics, and multilayer materials.

For a general type of surface, it is advisable to determine its roughness class (smooth or rough) relative to the parameter α .

It is suggested to use the Disney BRDF (formulas (6.3–6.4)) to represent the specular properties of rough surfaces of a general type. Disney BRDF includes the usage of the GGX distribution as D for the main specular lobe, which, compared to other known distributions, such as Beckmann, is characterized by narrower specular peaks and longer attenuation zones around them [5]. Due to this, the realism of object visualization increases and this model is the main one in modern three-dimensional graphics software, such as Blender.

Accordingly, to represent the anisotropic reflection of light from rough surfaces of a general type, it is advisable to use the anisotropic version of the D of Disney model.

Disney model has been proposed to represent the specular component of color, so it is also advisable to use the diffuse component of the Disney model (formula (6.5)). The diffuse Disney model, although simpler than the Oren-Nayar BRDF, is characterized by greater flexibility and a more accurate approximation of the measured sets of material reflectance values.

Next, we consider the case when the surface of the object is smooth. To represent the diffuse reflection of light, it is proposed to use the modified Lambert model (formula (6.2)). This model is widely used and does not take into account the surface roughness parameter. It is assumed that the microfacets of the surface are smaller than the scale of light scattering, so the surface is considered smooth.

For the specular component of the reflection model, it is proposed to use a simple BRDF that does not take into account the surface microstructure and, accordingly, the roughness parameter. If the surface roughness is low, it is possible to obtain a highly realistic image without unnecessary calculations of the distribution of microfacets.

Let us determine the level of surface roughness at which the use of empirical BRDF instead of microfacet BRDF does not lead to significant differences between the formed images.

For this purpose, we will consider the use of the energetically-correct Phong BRDF (F_{PHE}), energetically-correct Blinn BRDF (F_{BE}), energetically-correct cosine-quadratic BRDF (F_{2E}), energetically-correct cosine-power BRDF of degree 16 (F_{16E}). Let's also analyze the application of more complex empirical models that include the calculation of the Fresnel factor, for example, the energy-correct Gotanda BRDF (F_{GE}). Such models can be considered a special case of microfacet BRDF, where simple BRDF (for example, Phong) is used as the distribution of microfacets D , and $G=1$ (no self-shading of microfacets). In particular, the cases were considered for $D = F_{PHE} (F_{CTPHE})$, $D = F_{2E} (F_{CT2E})$, $D = F_{16E} (F_{CT16E})$, when $G=1$.

The study of the effect of replacing the microfacet BRDF with the empirical one at low surface roughness was carried out using the BRDF Explorer software tool. The study used the test figure "Robot" (Fig. 6.72).

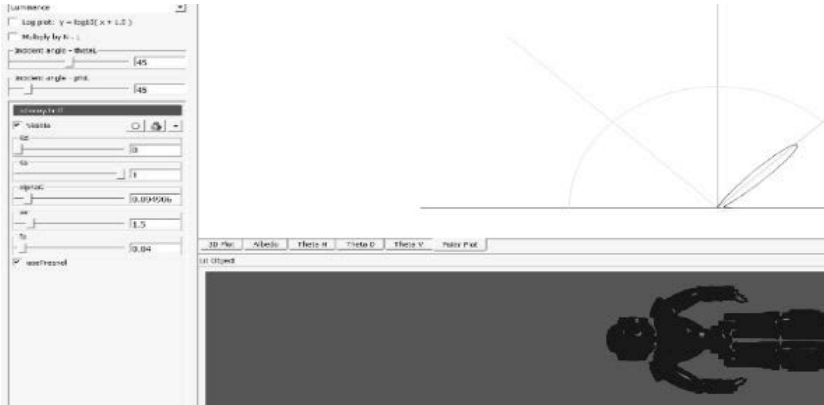


Fig. 6.72. Visualization of the test figure "Robot" in BRDF Explorer

For the specular component of the Disney BRDF, the following surface roughness α_g levels were considered: $1; \frac{9}{10}; \frac{8}{10}; \frac{7}{10}; \frac{6}{10}; \frac{5}{10}; \frac{4}{10}; \frac{3}{10}; \frac{2}{10}; \frac{1}{10}; \frac{1}{20}; \frac{1}{100}; \frac{1}{200}; \frac{1}{1000}; \frac{1}{2000}; \frac{1}{10000}; \frac{1}{20000}; \frac{1}{100000}$. An image of the test figure was generated for each specified roughness level.

The image based on each empirical BRDF candidate was obtained for the following values n : $1; 5; 10; 20; 40; 60; 80; 100; 200; 300; 400; 500; 600; 700; 800; 900; 950; 1000$.

For each roughness level, the smallest deviation between the image based on the microfacet model and the image based on the empirical model relative to the specified n values was determined. For this purpose, the NMSE [3] metric was used.

$$\frac{\sum_i (R_1(i) - R_2(i))^2 + (G_1(i) - G_2(i))^2 + (B_1(i) - B_2(i))^2}{\sum_i R_1(i)^2 + G_1(i)^2 + B_1(i)^2},$$

where R, G, B are the pixel color intensities for the RGB model channels respectively, i is the image pixel number, the index 1 corresponds to the reference image, the index 2 corresponds to the image based on a simpler light reflection model.

Table 6.8 shows the minimum NMSE values during the visualization of the test figure using the models F_{PHE} , F_{BE} , F_{2E} , F_{16E} .

Table 6.8

Minimum NMSE values during the “Robot” figure visualization using the models F_{PHE} , F_{BE} , F_{2E} , F_{16E} for particular level of surface roughness

α_g	α	Minimum NMSE (f_{PHE})	Minimum NMSE (f_{BE})	Minimum NMSE (f_{2E})	Minimum NMSE (f_{16E})
1.0	1.000	0.0554	0.0383	0.0517	0.0430
0.9	0.9487	0.0576	0.0341	0.0596	0.0427
0.8	0.8944	0.0589	0.0305	0.0577	0.0432
0.7	0.8367	0.0543	0.0283	0.0555	0.0421
0.6	0.7746	0.0542	0.0272	0.0558	0.0436
0.5	0.7071	0.0545	0.0268	0.0548	0.0452
0.4	0.6325	0.0491	0.0282	0.0472	0.0473
0.3	0.5477	0.0419	0.0275	0.0465	0.0508
0.2	0.4472	0.0330	0.0271	0.0445	0.0410
0.1	0.3162	0.0195	0.0252	0.0322	0.0264
0.05	0.2236	0.0099	0.0139	0.0190	0.0150
0.01	0.1000	0.0018	0.0027	0.0031	0.0028
0.005	0.0707	0.0008	0.0017	0.0015	0.0013
0.001	0.0316	0.0004	0.0012	0.0003	0.0003
0.0005	0.0224	0.0004	0.0011	0.0001	0.0002
0.0001	0.0100	0.0003	0.0010	0.00003	0.0002
0.00005	0.0071	0.0003	0.0010	0.00001	0.0002
0.00001	0.0032	0.0003	0.0010	0.000002	0.0002

Table 6.9 shows the minimum NMSE values during the visualization of a test figure using models F_{GE} , F_{CTPHE} , F_{CT2E} , F_{CT16E} .

Table 6.9

Minimum NMSE values during the “Robot” figure visualization using the models F_{GE} , F_{CTPHE} , F_{CT2E} , F_{CT16E} for particular levels of surface roughness

α_g	α	Minimum NMSE (F_{GE})	Minimum NMSE (F_{CTPHE})	Minimum NMSE (F_{CT2E})	Minimum NMSE (F_{CT16E})
1.0	1.000	0.0075	0.0109	0.0052	0.0047
0.9	0.9487	0.0050	0.0108	0.0042	0.0060
0.8	0.8944	0.0031	0.0113	0.0045	0.0087
0.7	0.8367	0.0020	0.0125	0.0063	0.0131
0.6	0.7746	0.0020	0.0142	0.0099	0.0194
0.5	0.7071	0.0037	0.0167	0.0159	0.0261
0.4	0.6325	0.0042	0.0204	0.0199	0.0273
0.3	0.5477	0.0061	0.0200	0.0221	0.0267
0.2	0.4472	0.0082	0.0185	0.0270	0.0176
0.1	0.3162	0.0096	0.0082	0.0159	0.0060
0.05	0.2236	0.0077	0.0042	0.0136	0.0076
0.01	0.1000	0.0017	0.0016	0.0032	0.0026
0.005	0.0707	0.0010	0.0008	0.0013	0.0012
0.001	0.0316	0.0009	0.0002	0.0002	0.0003
0.0005	0.0224	0.0009	0.0003	0.0001	0.0002
0.0001	0.0100	0.0009	0.0003	0.0001	0.0002
0.00005	0.0071	0.0009	0.0003	0.0001	0.0002
0.00001	0.0032	0.0009	0.0003	0.00004	0.0002

When determining the degree of visual differences between reference images and images based on simple light reflection models, Table 6.10 [3] was used. This table contains the correspondence between the NMSE value and the level of visual differences.

Table 6.10

NMSE value when comparing the original image with the reference image

NMSE value	Level of differences between images
< 0.0001	There are no visual differences.
0.0001–0.00025	There are minor visual differences
0.00025–0.001	There are noticeable visual differences
> 0.001	The images are significantly different

Therefore, at high and medium α_g roughness, the use of more complex light reflection models (see Table 6.9) provides the formation of images that are more similar to the reference ones. However, at small α_g roughness values, the losses in image visualization quality are approximately the same for simple empirical models (see Table 6.8) and empirical BRDFs based on microfacet models (see Table 6.9).

At the same time, the smallest value of NMSE (0.000002) is achieved when using the energetically-correct cosine-quadratic BRDF f_{2E} . At $\alpha_g = 0.001$ ($\alpha = 0.0316$) NMSE corresponds to noticeable, but insignificant differences between images, at $\alpha_g = 0.0005$ ($\alpha = 0.0224$) NMSE corresponds to minor differences between images, with $\alpha_g \leq 0.0001$ ($\alpha \leq 0.0100$) the differences between images are imperceptible.

Fig. 6.73 shows comparative images of the visualized test figure “Robot” when using the specular component of the Disney model ($\alpha_g = 0.1$, $\alpha_g = 0.001$, $\alpha_g = 0.00001$) and when using the energetically-correct cosine-quadratic model ($n = 800$, $n = 900$, $n = 1000$).

The shapes are visualized in BRDF Explorer.

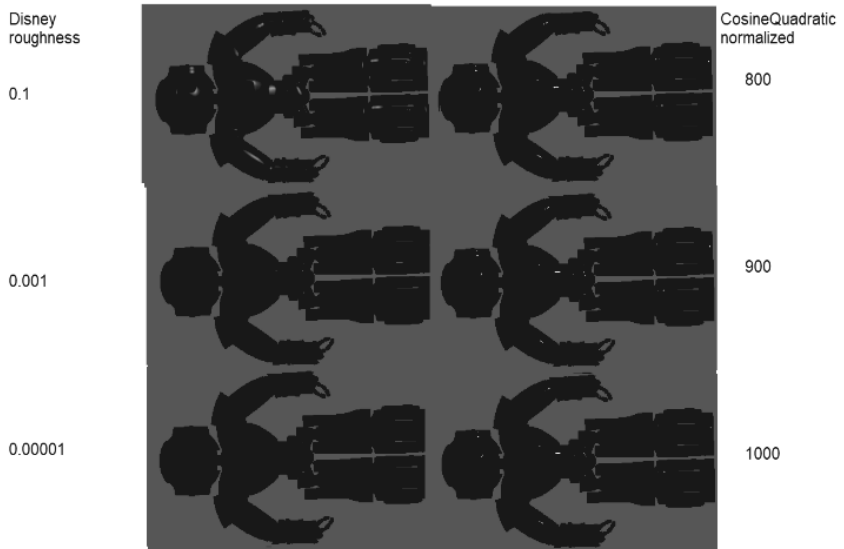


Fig. 6.73. Comparison of the results of visualization of a test figure based on the Disney specular BRDF and the energetically-correct cosine-quadratic BRDF for particular α_g , n levels

Therefore, to represent specular reflection of light from smooth surfaces ($\alpha \leq 0.03$), it is recommended to use the energetically-correct cosine-quadratic BRDF.

The diffuse and specular models of light reflection from a smooth surface are combined using the coefficients k_d, k_s .

For rough fabrics, such as wool and cotton, it is advisable to use the Neubelt-Pettineo microfacet BRDF [25], which has been established by practical experiments of scientists from the Naughty Dog company. This model is widely used, in particular by Ready at Dawn, Naughty Dog companies, and in the graphics renderer Filament.

For shiny fabrics such as silk and velvet, it is advisable to use the microcylindrical Kajiya-Kay BRDF [25]. Researchers from Naughty Dog has found [25] that using computationally more complex microfacet models provides a somewhat more accurate reproduction of the visual properties of materials. However, the advantage is in most cases imperceptible.

For hair, it is proposed to use Huang BRDF, which is one of the newest BRDFs of this type and is based on the components of Marschner BRDF, it is the basis of the hair reflection model in Blender.

To recommend the BRDF for multilayer surfaces, the results of experiments by M. Baty et al. [9] were taken into account, where the usage of the Weidlich-Wilkie BRDF and Belcour BRDF were compared. Plastics, transparent plates, and metals were considered as the main surfaces.

For plastics, it is advisable to use the Weidlich-Wilkie model [9], since the Belcour model is specular.

For transparent plates, it is advisable to use the Belcour model [9], since it is necessary to take into account multiple light scattering.

When modeling the illumination of metal surfaces, the Belcour model has been found to be more accurate [9]. The use of the Weidlich-Wilkie model is particularly problematic when a smooth metal is covered with a rough additional layer.

Thus, the algorithm is based on a preliminary definition of the reflective properties of materials and can be used both to provide high-performance rendering and photorealistic visualization of three-dimensional scenes. In the case of high-performance rendering, the method allows to choose the most productive model of the threshold level of accuracy. In the case of highly realistic rendering, the method provides an optimal choice of reflection models for different types of surfaces and eliminates unnecessary computational costs when visualizing smooth surfaces.

6.5. CONCLUSIONS

In the work, new bidirectional reflectance distribution functions were developed to increase the realism and productivity of glare reproduction on the surfaces of objects in 3D scenes.

A number of modified Schlick BRDFs are proposed, which provide increased realism of the glare attenuation zone compared to the original function. The first of them lies in introducing new coefficients and an additional power. The second of them lies in using the Schlick expression as a member of a cubic polynomial. The third function is the sum of the original Schlick function and the Schlick function with selected coefficients.

A combined function is proposed, which lies in using a cosine-power function of the 4th degree in the glare epicenter zone and a cosine-power function of the 16th degree in the glare attenuation zone. An increase in glare reproduction performance in the epicenter zone is provided while maintaining the appropriate level of realism. The second proposed combined BRDF lies in a combination of a quadratic function in the glare epicenter zone and a cubic function in the glare attenuation zone. As a result, the realism of the glare attenuation zone reproduction increases and the productivity of the glare epicenter zone reproduction is enhanced. The third proposed combined BRDF lies in increasing the realism of the glare blooming zone reproduction by combining two cubic functions.

The energetically-correct logarithmic BRDF, modified Schlick BRDF, and BRDF based on the calculation of the angle between vectors have been developed. These functions comply with the law of energy conservation and provide a highly realistic representation of light reflection from object surfaces.

An adaptive method for selecting BRDF is proposed, which allows to select the optimal function depending on the rendering goal and the optical properties of the surface.

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