

Interval fuzzy sets in the tasks of recognizing and predicting the states of complex objects in conditions of incomplete data

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ABSTRACT

In the tasks of recognizing and predicting the states of complex objects, a common situation is when it is necessary to make decisions with contradictory or incomplete data, which is caused by the presence of a certain level of “noise” or gaps in the data. To solve the existing problem, interval fuzzy sets, or fuzzy sets of type-2, are used, when the values of the membership functions are not a number, but an interval. The report shows the appearance of an interval output in fuzzy logic systems of type-2 when using interval membership functions and the construction of a set of fuzzy models of type-2 using set-theoretic operations for making effective decisions in applied problems.

Keywords: complex object, incomplete data, fuzzy set, interval membership function, fuzzy logic system of type-2

1. INTRODUCTION

Today's problems of decision-making in complex conditions occupy a special place in information technology. Mathematical methods have become widely used for the description and analysis of complex economic, social, medical and other objects. However, it should be noted that the main difficulties in such problems arise during the description of the objects when the set of object parameters is incomplete, and there is a need to conduct research on the influence of such a factor on the effectiveness of making a final decision in problems of this type. To overcome the difficulties of reflecting the insufficiently defined or incomplete data or concepts, as well as for the analysis and modeling the systems in which a person participates, the American mathematician Lotfi Zadeh suggested the theory of fuzzy sets in 1965^{1,2}. Further development of this theory showed its theoretical and practical significance within the framework of mathematical modeling of complex systems and objects that have insufficiently defined volumes of input data. However, methods that allow not only to recognize the states of complex objects in conditions of incomplete data using interval fuzzy models of type-2, but also to effectively estimate the interval at the output of the interval model have not been sufficiently studied.

2. PRELIMINARIES

It is known that interval fuzzy^{1,2} sets are characterized by the fact that the value of membership functions is not point, that is, in fuzzy logic systems for further calculations, not a number, but an interval is given. This approach allows us to represent the problems of recognizing the states of complex objects in conditions when it is necessary to make decisions, having contradictory or underdetermined data³.

In the work⁴, a method for constructing type-2 fuzzy logic systems with interval membership functions is presented, which will be used not only for medical diagnostics tasks but also for disease state prediction tasks. In the works^{5,6}, an approach to constructing type-2 interval fuzzy logic systems in the medical field based on an experimental data sample with gaps is proposed.

Mathematical information from type-2 interval fuzzy sets has been successfully used⁷. In this work, the mathematical apparatus based on fuzzy sets of type-2 was implemented in the process of modeling the states of a complex natural system under conditions of uncertain input data. As a subject area, the work presents the process of assessing the state of an artesian well within the limits from the beginning of hydrogeological exploration to its end. For the natural subject

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area in this formulation, the characteristic features are problems of access to experimental data, measurement inaccuracy and other factors. These factors contribute to the appearance of uncertain output data. The experimental studies conducted in the work showed that the implementation of the apparatus of fuzzy sets of type-2 allowed to obtain an effective solution to this problem.

Let us take into account that the use of the interval fuzzy set apparatus in the tasks of recognizing and predicting the states of complex objects in conditions of incomplete data should be accompanied by experimental studies and have a justification for the adequacy of fuzzy models built on the basis of type-2 fuzzy sets. Therefore, the works^{8,9}, demonstrate the studies on the adequacy in representing the subject area by type-2 fuzzy models built with the consideration of the uncertainty of the input data. Within the framework of the analysis of the subject area, we will adopt the following goal and objectives of the study. The goal of the study is to expand the possibilities in solving the tasks of recognizing and predicting the states of complex objects in conditions of incomplete data. Considering that fact in such a problem statement, the knowledge bases will contain incomplete information, therefore we will propose procedures for evaluating information at the output of the type-2 interval fuzzy model for making further effective decisions.

3. MAIN RESULTS

Currently, the relevance of data analysis tasks under uncertainty is increasing, namely the presentation and processing of fuzzy or incomplete data proposed by experts or obtained on the basis of experimental data processing. Fuzziness, which is characteristic of the initial data, usually arises when the expert tries to qualitatively characterize the quantitative concepts and relations that he needs in the process of solving the problem. For example, the expert uses natural language to describe tasks, from which linguistic uncertainty arises¹. This type of uncertainty is associated with the use of natural language to describe tasks and is generated by the ambiguity of the content of phrases and the multiplicity of meanings of words (concepts and relations) of the language chosen by a person. Difficulties in presenting data also arise when experts use imprecise data in their reasoning, that is, such values that were obtained using measurement examples, where there is definitely a pre-accepted measurement error. It should be noted that in the process of collecting information at some step it may turn out that only part of the information has been collected, or not all possible information has been provided, or not all necessary information has been provided. Such a fact is called incompleteness of the data; very often incompleteness is mainly due to the fact that the expert does not know the relevant factors or has not noted the necessary factors that are important for solving the problem^{2,8,9}.

We will consider the study of the possibilities, advantages and disadvantages of interval fuzzy sets used for recognizing and predicting the states of complex objects in conditions of incomplete data using a fuzzy model of type-2 with interval membership functions, which is shown in Figure 1.

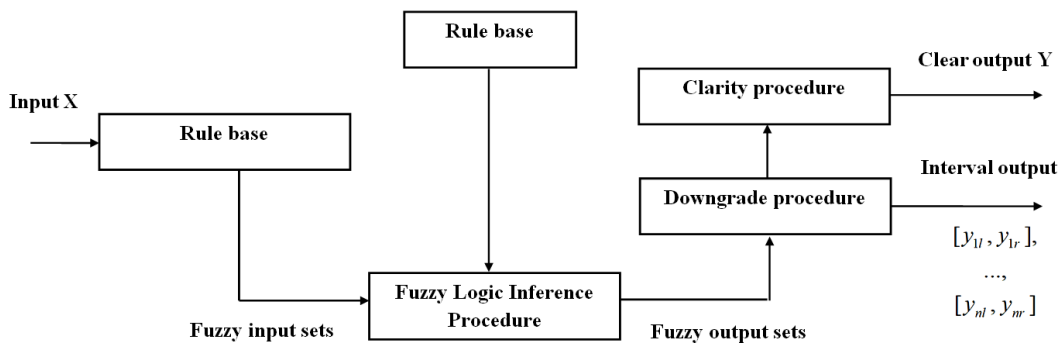


Figure 1. Type-2 fuzzy model

This mathematical model is an interval fuzzy model of type 2, which includes a rule base (fuzzy knowledge base), a fuzzification procedure, a fuzzy inference procedure, a type reduction procedure, and a definability procedure [2].

The presented interval fuzzy model of type-2 is generated based on an experimental sample, namely: a known experimental sample Q :

$$Q = \{Q_1, Q_2, \dots, Q_n\},$$

where $Q_i = (q_{1i}, q_{2i}, \dots, q_{ki}, y_i), i = 1 \dots n$; n is the number of experimental examples, k is the number of input variables, and y is the output value.

The input vector is a set of values of the object parameters $Q^i = \{q_1^i, \dots, q_m^i\}$. The rule base is formed from known pairs (Q_i, Y_i) , where the input vector is assigned a linguistic estimate of the value of the output variable Y . Thus, each input vector of experimental data generates one rule. The antecedents of the rules are formed by replacing the value q_j^i with the corresponding fuzzy term A_{qj}^i , the consequents are the term of the linguistic variable y , defined for the vector Q^i :

$$R^i : \text{ IF } q_1 \in A_{q1}^i \wedge q_2 \in A_{q2}^i \wedge \dots \wedge q_m \in A_{qm}^i \text{ THEN } y \in S_y^k \in \{S_1, \dots, S_p\},$$

where q_i denotes input variables, y the output variable, and $S_y \in \{S_1, \dots, S_p\}$ the term sets of the output variable.

As the membership function for the input variables, we take a Gaussian function of type²

$$\mu(x) = e^{-\left(\frac{x-b}{c}\right)^2}$$

When constructing an interval fuzzy model from experimental data, the membership functions of the input variables of the Gaussian type will change according to the algorithm⁵. Taking into account the conditions of incomplete data, namely the case when the input vector may contain gaps, the use of this algorithm will lead to the possible appearance of an uncertainty zone in the interval membership function^{10,11,12}. This zone will expand depending on the existing incompleteness of the input data. The general form of such an interval membership function:

$$\mu(x) = e^{-\left(\frac{x-b}{[\min(c), \max(c)]}\right)^2},$$

where $[\min(c), \max(c)]$ is the range of change of the Gaussian membership function parameter, shown in Figure 2.

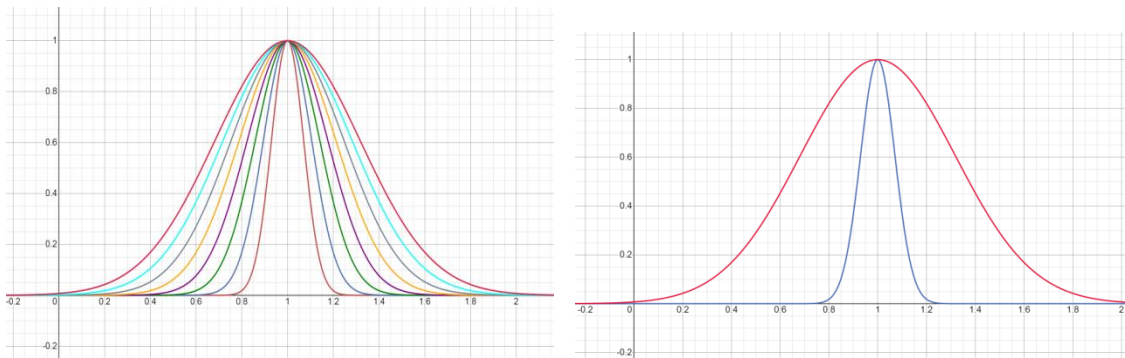


Figure 2. Range of change of the Gaussian membership function parameter

We present the results of the functioning of a type-2 fuzzy model, which is built using interval membership functions under conditions of incomplete data in the task of recognizing and predicting the states of a complex object. As a first example, we consider an interval fuzzy model, which was built to recognize the states of endocrine system diseases from different sets of input data^{5,6,7}.

Fragment of experimental research (first example).

Knowledge about the subject area is characterized by the following parameters:

- 39 input variables;
- one output value - a conclusion about the absence or presence of an endocrine system disease with a definition of the degree of severity; from 1 to 4.
- 140 experimental examples.

The output of the model is interval. The results of the model are shown graphically using the state diagram in Figure 3.

Situation A of the state diagram corresponds to the interval output of the fuzzy model $y = [0.9560; 1.1431]$. The values of the upper and lower limits of the interval correspond to the same diagnosis - the first state of the disease, or the initial degree of severity of the disease. Conclusion: the state has been recognized, that is, the first state that characterizes the object has been established¹³⁻¹⁵.

Situation B of the state diagram shows the interval output of the fuzzy model $y = [0.5727; 3.287]$. The values of the upper and lower limits of the interval correspond to different states of the disease. Conclusion about the recognition and prediction of the disease state: only the prediction of the states has been performed, that is, in this case, the final result will be all the states that fall into the output range.

Situation C of the state diagram corresponds to the interval output of the fuzzy model $y = [2.13373; 2.4341]$. The values of the upper and lower limits of the interval correspond to the same diagnosis - the second state of the disease. Conclusion about the recognition and prediction of the disease state: the state has been recognized, that is, the second state that characterizes the object has been established.

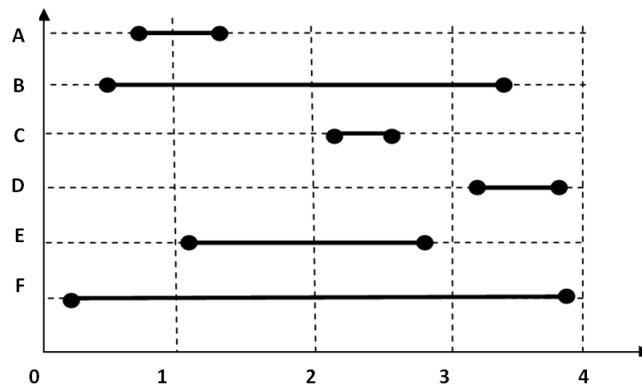


Figure 3. State diagram

Situation D of the state diagram corresponds to the interval output of the fuzzy model $y = [3.0427; 3.7675]$. The values of the upper and lower limits of the interval correspond to different disease states. Conclusion about the recognition and prediction of the disease state: only the prediction of states occurred, that is, in this case the final result will be all states that fall into the output range, the final decision is made by an expert in this field of study. Situation E is considered similarly.

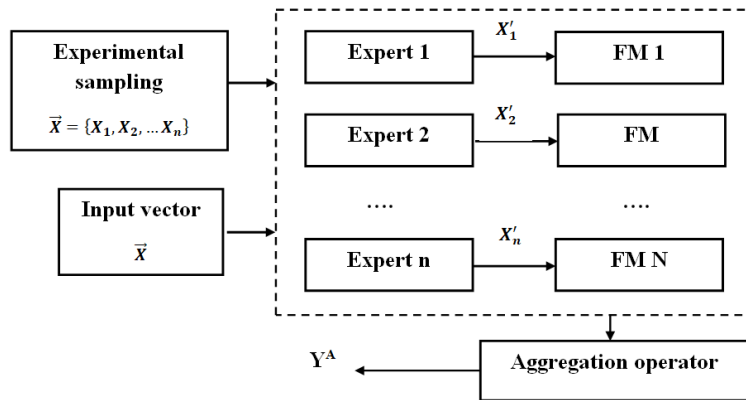


Figure 4. Structure of the set of interval fuzzy models with the aggregation operator

The general conclusion for the specified states of the object can be made as follows: the results of the operation of the interval fuzzy model of type-2 adequately reflect the subject area under conditions of incomplete data. But in the case of situations B and D, several states fall into the range, and the final conclusion about the prediction of states belongs to the expert. In the case of situation F, when we record the appearance of an extended interval, that is, all possible states are

present at the output, it should be noted that the initial data is not enough to recognize and predict the state of the disease and additional research is required.

Thus, in order to improve the solution at the output of the interval fuzzy model of type-2 in situations similar to the statement *F*, we will propose an approach to solving the problems of recognizing and predicting complex objects. As such a solution to the problem in this statement, we will construct several interval fuzzy models of type-2, in each of which experts will offer a set of input features selected by each expert. According to the experts, each interval fuzzy model of type-2 will give its own interval at the output. Comparing the intervals by set-theoretic operations, which are implemented by the aggregation operator, will give the initial result as a solution to the problem. The set of fuzzy models of type-2 proposed by the experts is shown in Figure 4.

Theoretically, set operations consist in using the intersection operation between the intervals of several fuzzy models, if it exists (shown in Figure 5) and the union operation of intervals, if the given models do not have it (shown in Figure 6). The final output of the fuzzy set model, which is the result of the intersection or union of the intervals of individual fuzzy models, is created as a result of the implementation of the aggregation operator Y^A in the form of a combination of set theory operations, namely: $U/\cap$.

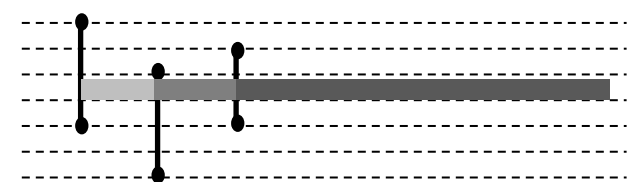


Figure 5. Fuzzy models have a total interval

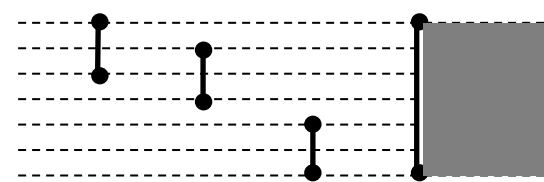


Figure 6. Fuzzy models do not have a common interval

As a second example, let us consider an interval fuzzy model of type-2, which was built to recognize states in a problem of a natural nature, where the object is an artesian well [7].

Fragment of experimental research (second example).

Knowledge about the subject area is characterized by the following parameters: we take into account 38 input variables and one output value: “well prospectivity”, which has, in addition to the linguistic variable, a scale of values from 0 to 10. The scale is divided into linguistic terms, such as: “insufficient prospectivity” - from 1 to 5 points; “sufficient” - from 6 to 7 points and “high” - from 8 to 10 (A fragment is shown in Table 1). The output of the model is interval. A fragment of experimental research is given at three stages of geological exploration (GE) (Table 2).

Table 1. Fragment of the work of the interval fuzzy model of the deposit with the assessment of one expert in conditions of incomplete data

№	Stage (GE)	Determined features/total	output	Expert assessment – well prospects
1	1	10/38	[1.69; 3.12]	1/ Insufficient (I)
	2	11/38	[1.51; 2.01]	
	3	11/38	[3.11; 3.76]	
2	1	24/38	[5.68; 6.32]	5/ Sufficient (S)
	2	24/38	[5.59; 7.26]	
	3			
3	1	9/38	[6.41; 7.74]	8/ Sufficient or High (S, H)
	2	16/38	[5.85; 8.58]	
	3	32/38	[6.23; 8.47]	
...				

For this problem, the aggregation operator GE performs the union operation (stage 1, 2) and the intersection operation (stage 3). Thus, the final interval output, which is the solution to the problem, will be determined by the result of executing the aggregation operator on the set of interval fuzzy models created by experts.

Table 2. Results of the interval fuzzy model in conditions of incomplete data with estimates from two experts
Feature/Total Definition Model Output

№ Stage (GE)	Expert 1 (FM1)		Expert 2 (FM2)		Aggregate output
	Feature/Total Definition	Model Output	Feature/Total Definition	Model Output	
1	9/38	[1,39;1,51]	8/41	[0,43; 0,45]	[0,43; 1,51]
2	16/38	[0,24;0,27]	19/41	[0,04; 0,04]	[0,04; 0,27]
3	39/38	[0,09;3,97]	41/41	[0,43; 3,97]	[0,43; 3,97]

4. CONCLUSIONS

The use of the interval fuzzy sets is proposed in the tasks of recognizing and predicting the states of complex objects under conditions of incomplete data. To solve the existing problem, interval fuzzy sets, or fuzzy sets of type-2, are used, when the values of the membership functions are not a number, but an interval. Within the framework of this approach, a fuzzy model of type-2 with interval membership functions and a set of interval fuzzy models are presented, where the use of the aggregation operator of set-theoretic operations enables us to obtain a solution to the problem. Experimental examples of research for applied problems are presented.

REFERENCES

- [1] Zadeh, L.A., “Fuzzy sets as a basis for theory of possibility”, Fuzzy sets and systems 100 supplements, 9-34. (1999).
- [2] Mendel, J.M., John, R.I., Liang, Q., “IntervalType-2 fuzzy logic systems: theory and design”, IEEE Transactions on Fuzzy Systems, 8,535-550 (2000).
- [3] Baryshev, Yuriy, Kondratenko, Natalia, Kazmirevsky, Vitaliy, Kyrylashchuk, Tetyana. “Fuzzy sets of type-2 in the problems of modeling and estimating the states of critical systems with underdetermined input data and using experts”, Information Technologies and Computer Engineering, Volume 57. No. 2, 13-24. (2023).
- [4] Kondratenko, N., “Interval Fuzzy Modelling of Complex Systems under Conditions of Input Data Uncertainty”, Eastern-European Journal of Enterprise Technologies, V.4/4 (82), 20–28 (2016).
- [5] Goguen, J.A. L-fuzzy sets. J. Math. Anal. Appl. 18, 145–174 (1967).
- [6] Chang, C. Fuzzy topological spaces. J. Math. Anal. Appl. 24, 182–190. (1968).
- [7] Rosenfeld, A. Fuzzy groups. J. Math. Anal. Appl. 35, 512–517 (1971).
- [8] Nasir, Abdul, Naeem Jan, Abdu Gumaei, Sami Ullah Khan, and Fahad R. Albogamy.. "Cybersecurity against the Loopholes in Industrial Control Systems Using Interval-Valued Complex Intuitionistic Fuzzy Relations" Applied Sciences 11, no. 16: 7668 (2021).
- [9] Eklund, Patrik, Jari Kortelainen, and Magnus Löfstrand. "Quantaes for Fuzzy Sets and Relations of Higher Types" Mathematics 13, no. 13: 2159, (2025).
- [10] Ushenko, Y.A., Dubolazov, A.V., Angelsky, A.P., Sidor M. I., et al., "Laser polarization fluorescence of the networks of optically anisotropic biological crystals", Proc. SPIE 8698, 869809 (2012).
- [11] Romanyuk, A.N., et al., “Fast ray casting of function-based surfaces”, Przegląd Elektrotechniczny, 93(5), pp. 83–86 (2017).
- [12] Wójcik, W., et al., “Medical Fuzzy-Expert System for Assessment of the Degree of Anatomical Lesion of Coronary Arteries”, Int. J. Environ. Res. Public Health, 20, 979 (2023).
- [13] Grądz, Ż., Wójcik, W., Gromaszek, K., Kotyra, A., Smailova, S., Isakova, A., Yeraliyeva, B., Kumargazhanova, S. and Imanbek, B. “Application of Fuzzy Neural Networks in Combustion Process Diagnostics,” Energies 17(1), 212 (2024). <https://doi.org/10.3390/en17010212>
- [14] Pavlov, S. V. at al., “Multichannel system for recording myocardial electrical activity”, Information Technology in Medical Diagnostics II, London : CRC Press, Balkema book, Taylor & Francis Group, 307–314 (2019).
- [15] Poplavskiy, O., Pavlov, S., et al. “High-performance information technology for processing large datasets and biomedical images to improve the accuracy of computer-aided decision support systems,” Proc. SPIE, vol. 12844, (2024); doi: 10.1117/12.3057444.