Regression method for inverse correlation filters design for objects recognition

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ABSTRACT

The problem of inverse correlation filters design to recognize a set of objects is considered as the problem of regression parameters estimation on the base of input data arrays and desirable response. The data and response should be processes with zero mean to consider this problem as evaluation of regression parameters. The problem is solved using the least squares method with regularization. The regularization is optimized to achieve high resolution of the filters in conjunction with capture' broad band of objects given by a set of templates. The least squares method is using in the terms of singular value decomposition that made it possible to linearize the nonlinear ridge regression optimization problem. The methods to false recognitions elimination are considered, It was shown that the regression approach gives additional condition to recognize classes of objects. This allows to have more high accuracy in recognition of desired objects on a foreign background in comparison with other correlation filters types.

Keywords: regression, objects recognition, correlation filter, inverse filter, optimized regularization

1. INTRODUCTION

Correlation filters (CF) are widely used in automatic image recognition and object tracking systems[1-3]. The CF have high resolution, which manifests itself in a high-quality response signal to a signal with the structure of the tracked object. The CF are implemented using a linear convolution operation, which modern processors perform efficiently. Images of objects can be recognized by CF as a combination of recognizable typical elements, such as Gabor and Haar functions, or as an integral object defined by its own special filter in the form of a vector or matrix[4-6].

There are several approaches to the development of CF, which differ in determining of a structure of the source and response data, a method to solve the problem of matching the input data with the response, additional conditions and restrictions. The task of matching initial data with a response is an inverse problem with weak conditioning, since there is much more data with restrictions and additional conditions than response data. The main method for solving it is the least squares problem. Since the linear convolution operation is invariant to the spatio-temporal shift of the source data, the second method is to use the Fourier transform. Both methods have disadvantages associated with inverting matrices of incomplete rank and inverting small values of spectral components[7-9].

The disadvantages of these methods increase significantly when the task is to develop a bank of filters for recognizing slightly different objects in shape, size and color. Also, the resolution of correlation filters is reduced under the influence of the background on which the target object is located. In this case, the object can be recognized by a small feature fragment. In [5-6], an inverse correlation filtering method was proposed and investigated to solve such problems. The method allows to obtain a set of quasi-orthogonal filters for recognizing a certain set of objects using selected feature fragments from sets of target image samples. It was considered its optimality to problems of objects recognizing solution and it was investigated some regularization approaches to least squares problem solution. The method works with vector and tensor data structures. Optimal high-resolution, low-error filters were obtained through an iterative training process of more than one hundred steps[10-12]. In addition, object recognition is used in various fields of science[13-18].

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A set of fragments of template images of *P* objects for training of *P* CF can be presented as the tensor[6] $X_{i_1,i_2,i_3,i_4,i_5,i_6,i_7}$, where i_1-th object is specified by $N_{T_{i_1}}$ templates that are indexed by $i_2 = 0....N_{T_{i_1}} - 1$, i_2-th template of i_1-th object includes $N_{F_{i_1,i_2}}$ overlaped fragments of size $N \times N$ pixels with indices i_3, i_4 for rows and columns, $i_5, i_6 = 0....N-1$ – indices of pixels of a fragment, $i_7 = 0,\ 1\ ,2$ – index of RGB colors. The object is recognized by the way of analysis of the spectrum of *P* responses of *P* filters with characteristic vectors $F = [\mathbf{f}_i]_{i=0,...P-1} = \lfloor f_{i,m} \rfloor_{i=0,...,P-1; m=0,...,3N^2-1}$.

The response of i_1-th filter for (i_3, i_4)-th fragment of the i_2-th template of *k*-th object is the next,

$$\sum_m \mathbf{x}_{k,i_2,i_3,i_4,m} f_{i_1,m} = \xi_{k,i_1,i_2,i_3,i_4}, \quad (1)$$

where summation is assumed over the domain of definition of the vectors values. A fragment of an image will be recognized as i_1-th object by responses spectrum $[\xi_m]_{m=0...P-1}$ in the basis of vectors F on condition[19-20]

$$\underset{i}{argmin} \sum_m \left| \delta_{i,m} - \xi_m \right| = i_1. \quad (2)$$

The task to design of the bank of P inverse correlation filters (ICF) can be formulated by joining of expressions (1) with averaging of objects' fragments relatively to desirable filters' responses as the following[19-21]

$$\frac{1}{N_{TF_k}} \sum_i \sum_{i_2,i_3,i_4} \mathbf{x}_{k,i_2,i_3,i_4,i} f_{i_1,i} = \delta_{i_1,k}, \quad (3)$$

where $\lfloor \mathbf{x}_{k,i_2,i_3,i_4,i} = vec_{i=i_5,i_6,i_7}\left(X_{k,i_2,i_3+i_5,i_4+i_6,i_7}\right) \rfloor_{i=0...3N^2-1}$, is the vector of lexicographic representation of the *k*-th object image fragment, N_{TF_k} is the total number of fragments of *k*-th object' templates, *i* is the index of the filter' samples and of the lexicographic representation of items indexed by i_5, i_6, i_7, $\delta_{i_1,k} = 1$: $i_1 = k$; $\delta_{i_1,k} = 0$: $i_1 \neq k$.

The filters responses in (1) differ from desirable ones in (3) to the same extent as individual fragments of objects' images differ from averaged fragments over a set of templates. The filters inputs and responses can be centered by means elimination to stabilize the filter responses in (1) and to obtain robust object recognition using rule (2). In this case, expression (1) can be considered as a regression model. Therefore, expression (3) must also be modified so that the inputs and responses in the right part will be zero-mean processes[17-19].

2. RELATED WORKS

The CF design was interpreted as a regression problem in [1-2]. However, when a problem of type (1) was formulating, the condition of zero mean values of the input image signal and the filter response, like in (2), was not specified. The response function is usually represented as a Gaussian function or a similar to Heaviside function. The first one has a maximum relative to the data of the desired object and a minimum relative to foreign objects, the second one, accordingly, has the values of one and zero[20-24].

The regression method is using in CF designing to solve the problem of filters characteristics finding as a coefficients of the ridge regression which optimize the functional[23]

$$\underset{f_{i_1},\cdot}{argmin} \sum_{i_1,k} \left\{ \left| \frac{1}{N_{TFk}} \sum_i \sum_{i_2,i_3,i_4} \mathbf{x}_{k,i_2,i_3,i_4,i} f_{i_1,i}(\lambda) - \delta_{i_1,k} \right|^2 + \lambda \cdot \sum_i f_{i_1,i}(\lambda) f_{k,i}(\lambda) \right\}, \quad (4)$$

where λ is the parameter of the regularization.

The methods of solving the optimization nonlinear problem like (4) were considered in [1-4, 7-9]. One of them is basing on using the singular value decomposition (SVD) [5-9] of the matrix compiled by input data to find its inverse manner under condition its ill-posed definition. The regularization procedure of the inversion of an ill-conditioned matrix by SVD consists in varying singular values so that they are not in the vicinity of zero.

3. THE REGRESSION METOD PROBLEM STATEMENT

The SVD of the matrix compiled by P columns

$$\mathbf{X} = \left[\frac{1}{N_{TFk}} \sum_{i_2,i_3,i_4} \mathbf{x}_{k,i_2,i_3,i_4,i} = \sum_{m=0}^{P-1} u_{i,m} v_{k,m} s_m \right]_{i=0...3N^2-1;k=0...P-1} = \mathbf{U} \cdot diag[\mathbf{s}] \cdot \mathbf{V}^T \tag{5}$$

,

where $v_{k,m}$ are elements of the unitary matrix $\mathbf{V}$ of size $P \times P$ with orthogonal columns: $\mathbf{V}^T\mathbf{V} = \mathbf{I}$, $u_{i,m}$ are elements of the similar matrix $\mathbf{U}$ of size $3N^2 \times 3N^2$ and s_m are P elements of the vector $\mathbf{s}$ of singular values: $s_m \geq s_{m+1} ... \geq 0$, T is the transposition of matrix, $diag[\cdot]$ is a matrix' diagonal, $\mathbf{I}$ is the identity diagonal matrix. The filters vectors F in (4) can be found from the condition that the partial derivative of expression (5) with respect to the filter vector is equal to zero. The using of data vectors presentation (5) gives the following solution in the vector-matrix form $F \cdot \mathbf{X} \cdot \mathbf{X}^T - \mathbf{X}^T + \lambda \cdot F \cdot \mathbf{I} = 0$, which in the terms of the SVD (5) looks as $F \cdot \mathbf{U} \cdot diag[\mathbf{s}^2] \cdot \mathbf{U}^T - \mathbf{V} \cdot diag \cdot [\mathbf{s}] \cdot \mathbf{U}^T + \lambda \cdot F \cdot \mathbf{I} = 0$. In this expression the second term can be moved to the right side and then both sides can be sequentially multiplied on the right by $\mathbf{U}$, $diag[\mathbf{s}^{-2}]$ and $\mathbf{U}^T$, then

$$F \cdot (\mathbf{I} + \lambda \cdot \mathbf{I}) = \mathbf{V} \cdot diag \cdot [\mathbf{s}^{-1}] \cdot \mathbf{U}^T \tag{6}$$

As the parameter of regularization λ is an arbitrary small positive value, expression (6) may be rewritten in the equivalent manner as the next one.

$$F = \left[f_{i,k}(\lambda) \cong \sum_{m=0}^{P-1} v_{\cdot i,m} u_{\cdot k,m} \left(s_m + \lambda\right)^{-1} \right]_{i=0...P-1;k-0...3N^2-1} \tag{7}$$

As it follows from compasizon of expressions (4) and (7), the using of the SVD allows to use for regularization a norm of data vector instead the norm of filter vector that linearising the optimization problem. This approach was used in [5-6] to optimize the solution of the least squares problem based on the balance of the relative variations of the original and inverted matrices, perturbed by the regularization of their singular value decomposition. The optimization procedure (4) in this case comes down to determining the optimal value of the regularization parameter at which the required filter quality is ensured.

The problem of the CF design is to compile a system of equations (1) with using only that images fragments that can serves as objects features. The training process allows to select the images data fragments in (1) which give true recognition of the objects by the previously defined estimation of the filters characteristics. These fragments can be concidered as objects features. The selected fragments are using for evaluation the filters characteristics in (3), (4).

As it was mentioned in [5-6], the properties of filters in terms of selectivity and capture band of object samples depend on the method of solving equations (3), (4), especially on the regularization method. It is necessary to take into account random variations in object image samples when determining filters to achieve stable recognition conditions with using (2). The input and response data in (3) and (4) must be zero-mean processes in this case[9]. The regression model of the ICF looks as the next:

$$\frac{1}{N_{TF_k}} \sum_m \sum_{i_2,i_3,i_4} \left(\mathbf{x}_{k,i_2,i_3,i_4,m} - \frac{1}{3N^2} \sum_n \mathbf{x}_{k,i_2,i_3,i_4,n} \right) \cdot f_{i,m} = \begin{cases} 1 : i = k \\ -1/(P-1) : i \neq k \end{cases} = \zeta_{i,k}. \tag{8}$$

The purpose of this work is to investigate how the formulation of the problem (1) in the form of multichannel regression (8) affects on the properties of the ICF: on the dynamics of the training process; on the error level; on the capture band of objects' templates' fragments.

As it follows from (7), the vectors of filters characteristics are columns of the matrix that is inverse in respect to matrix (5) with perturbated singular values. Therefore the filters like (5) are inverse in respect to input data.

4. THE REGRESSION REGULARIZATION

The optimal solution to the problem of determining the characteristics of objects filters as regression coefficients can be found using the functional

$$J(\lambda) = \underset{\lambda}{argmin}\left\{\sum_{i,k}\left|\sum_{m}\tilde{\mathbf{x}}_{k,m} f_{i,m}(\lambda) - \zeta_{i,k}\right|^2\right\} \tag{9}$$

where $\tilde{\mathbf{x}}_{k,m} = \frac{1}{N_{TFk}}\sum_{i_2,i_3,i_4}\left(\mathbf{x}_{k,i_2,i_3,i_4,m} - \frac{1}{3N^2}\sum_{n}\mathbf{x}_{k,i_2,i_3,i_4,n}\right)$, $\mathbf{f}_i(\lambda)$ is perturbated by regularization filters' vector. In terms of SVD (5), (7) functional (9) is as follows $J(\lambda) = \underset{\lambda}{argmin}\left\{\sum_{i,k=0:}^{P-1}\left|\sum_{m=0}^{P-1}\frac{v_{i,m}v_{k,m}s_m}{(s_m+\lambda)} - \zeta_{i,k}\right|^2\right\}$.

A zero value of the partial derivative of it with respect to the regularization parameter defines the condition of optimal value of the regularization parameter

$$J_\lambda(\lambda) = \frac{\partial J(\lambda)}{\partial\lambda} = -2\sum_{i,k=0:}^{P-1}\left|\sum_{n=0}^{P-1}\frac{v_{i,n}v_{k,n}s_n}{(s_n+\lambda)} - \zeta_{i,k}\right|\cdot\sum_{m=0}^{P-1}\frac{v_{i,m}v_{k,m}s_m}{(s_m+\lambda)^2} = 0. \tag{10}$$

As it follows from (10), the functional (9) is not limited by variable λ: $J(\lambda\to\infty)\to 0$. The energy of perturbated by regularization input data matrix increases indefinitely under this condition. Therefore, the functional (9) should be supplemented with a functional that limits the energy of the data matrix, which in terms of SVD (5) looks as:

$$I(\lambda) = \underset{\lambda}{argmin}\left\{\sum_{i,k=0:}^{P-1}\left|\sum_{m=0}^{P-1}v_{i,m}v_{k,m}(s_m+\lambda)^2\right|^2\right\}. \tag{11}$$

Its derivative

$$I_\lambda(\lambda) = 4\sum_{i,k=0:}^{P-1}\left|\sum_{m=0}^{P-1}v_{i,m}v_{k,m}(s_m+\lambda)^2\right|\cdot\sum_{n=0}^{P-1}v_{i,n}v_{k,n}(s_n+\lambda). \tag{12}$$

The balanced joining of the functionals (9) and (11) relative variations was used in [5-6]. In the considering case, due to the orthogonality of the vectors $v_{\cdot,m}$, the $J(0)$ becomes trivial, so it can be applied the balance of derivatives (10) and (12) with using their unperturbated values. Then they can be joined and the optimal λ_{opt} is provided by the condition of balanced relative variations of errors squares in (9) and data energy (11), such as

$$\frac{I_\lambda(\lambda)}{I_\lambda(0)} + \frac{J_\lambda(\lambda)}{J_\lambda(0)} \approx 0\,. \tag{13}$$

The desirable value of λ_{opt} can be found numerically from (13) by iterative scheme with initial value of λ_{opt} equal to minimal non zero singular value in (5). The solution of the problem (9) with account of λ_{opt} is the following.

$$\left[f_{k,i}(\lambda_{opt}) = \sum_{m,n=0}^{P-1}v_{\cdot i,m}u_{\cdot n,m}\left(s_m+\lambda_{opt}\right)^{-1}\varsigma_{k,n}\right]_{k=0...P-1;i-0...3N^2-1}. \tag{14}$$

The regression model (8) gives the filters (14) bank which has the property

$$\sum_{i_1=0}^{P-1}\sum_{i=0}^{3N^2-1}\tilde{\mathbf{x}}_{k,i} f_{i_1,i} \cong 0: \quad k = 0...P-1. \tag{15}$$

for all objects of the considering class. This property is due to the shape of the response signal in (8).

5. EXPERIMENTAL ANALYSIS

Typically, correlation filters are trained using templates of objects, but when target objects are recognizing in their environment a shape of fragments of a background can manifest itself as desired objects, which leads to many errors. Therefore, the goal of this experimental analysis is to compare the accuracy and capture bandwidth of correlation filters obtained by solving regression problems with different optimization formulations at recognizing of templates and objects on a background. Analysis of an effectivity of the ICF was made using 2600 templates images of 35 characters of the size from to pixels, cropped from 280 car license plate images, these images were used as the backgraund at the characters reconition. Up to 300 thousands fragments of size, were used for recognising the teplates and objects. The basis of quasi-orthogonal filters was obtained by solving the ill-posed problem (8) by inverting a rectangular data matrix. An accuracy of solving such problems depends from size of the data matrix. Therefore, an estimate of the recognized object (2) with using the spectrum (1) in the filter basis should be checked in filter bases obtained for a smaller number of objects[5].

Let, then the objects can be separated on T groups of size Q. The filters (14) can be defined for each of T groups using the pseudoinversion of the matrices of vectors which corresponds to frames of the objects group. Also, can be defined filters for Q groups by T objects. Then the maximal spectral element in (2) should be maximal element of the corresponding spectral groups of Q and T objects which include this object. The combination of spectrums in full basis and in groups' basises allows to reduce the error level by 20-25%. In the considered case P=35, T=7, Q=5.

The training process allows to select fragments of objects that have a spectrum (1) that ensure correct recognition of the object according to rule (2). Also, can be selected fragments that are recognized as erroneous and exclude them by filtering using filters basis similar to (8). The basises for true and false recognitions can be joined in (9) and then it will be defined the extended basis of filters $\left[F^{true}, F^{false}\right]$. It is obvious, the basises of true and false recognitions are mutualy orthogonal. As experimental investigations have shown, the use of the extended spectrum of true and false objects can reduce the level of recognition errors by 15-20%.

Table 1. Dynamic of training process for ICF (14) with optimized regularization from condition of balanced relative variations (13) for fragmets size (high), (low)

Training step	Feature fragments	True fragments	False fragments	True rating
0	1136 1566	762 1127	364 439	0.6707 0.7196
5	627 1006	535 869	92 137	0.8532 0.8638
10	535 884	468 792	85 92	0.8462 0.8959
20	554 832	486 781	68 51	0.8772 0.9387
30	443 811	405 770	38 41	0.9182 0.9494
40	428 800	393 761	35 39	0.9182 0.9511
50	452 813	408 784	44 29	0.9026 0.9643

Table 2. Dynamic of training process for ICF (14) with optimized regularization from condition of balanced relative variations[5] for fragmets size 11×11 (high), 13×13 (low)

Training step	Feature fragments	True fragments	False fragments	True rating
0	3255 2909	1857 1960	1398 949	0.5705 0.6737
5	2618 2830	1690 1910	928 920	0.6455 0.6749
10	2449 2546	1574 1794	875 752	0.6427 0.7046
20	2237 2345	1417 1708	820 637	0.6334 0.7283
30	2231 2267	1407 1713	824 554	0.6385 0.7556
40	2277 2233	1454 1700	823 533	0.6385 0.7613
50	2268 2165	1457 1677	811 488	0.6424 0.7745

The tensor of spectrum (1) amplitudes was accumulated at each t-th step of training process $\Xi^{t}_{i,k,m} = \left(\Xi^{t-1}_{i,k,m} + \xi_{i,m}(k)\right)/(N_{i,k}+1)$; $N_{i,k} = N_{i,k}+1$ where $\xi_{i,m}(k)$ is the spectrum (1) of i-th object that is recognized as k-th object, $N_{i,k}$ – number of recognitions. This tensor allows to selectc true amplitudes spectrum for i-th objects, that was previously identified by spectrum combination, by the rule:

$$\underset{k}{minarg} \sum_{m=0}^{2P-1} \left|\xi_{i,m} - \Xi_{i,k,m}\right| \begin{matrix} = i, \ object\ is\ true \\ \neq i, \ object\ is\ \ false \end{matrix}, \tag{16}$$

The rule (16) eliminates errors caused by the fact that different objects contain similar fragments.

The condition (15) was used for extended filters spectrum in normalized form

$$\rho(\xi) = \frac{\sum_{i=0}^{2P-1} \xi_m}{\sum_{i=0}^{2P-1} \left|\xi_m\right|}. \tag{17}$$

The dynamic process of training the filter (14) is presented in Table 1. The P filters of full basis and $2P$ filters of narrowed basises for spectrums combination are using in the initial step. Then, it is using additional basis of filters of false objects recognition. Also, the data of the dynamics of filters (14) training with regularization using the method of balanced relative variation cosidered in[5] are presented in Table 2. As it follows from the tables data, the filters quickly achieve recognition efficiency, but the efficiency depends on the fragment size. The filters reache a certain maximum efficiency in the number of captured object fragments and in the number of errors. the efficiency depends on the level of regularization, the larger λ in (14), the wider the band of fragments capture and the greater the number of errors. Therefore, it is important to determine the level of optimal regularization λ_{opt}, in this case using (13).

The training process produces a bank of filters with a narrow band of capturing objects fragments with a low error level. This filter bank creates a filtering branch. Fragments of objects not captured by the filter bank can be used to create the next recognition branches. By this way five branches were created to capture 95% of the objects templates.

Table 3. Comparison of IRF with Correlation Filters on recognizing of character objects on an arbitrary background

Filter type	Recognized fragments	True fragments	False fragments	True rating
RIFF	2958	2826	132	0.9553
DIRF	3187	2943	244	0.9234
MECF	5218	4156	1062	0.7964
ZACF	4764	3982	782	0.8358

The branches of true and false objects' fragments filtration create the convolutional system. This convolutional system was used to recognize character objects on an arbitrary background. The recognizing were made with using ICF (14) with regression response in (8) – RICF, ICF[5] with response as delta-function in (4) – DICF, Minimum Energy Correlation Filter (MECF)[1] and Zero-Aliasing Correlation Filter (ZACF) [3]. The results of recognizing are presented in Table 3. The table data showed that RICF has a minimal error rate. This was also achieved as a result of applying the spectrum selection condition (17) for subsequent analysis of spectral combinations and amplitude spectrum.

Statistical analysis during the training process showed that condition (17) can be used as $\rho(\xi)<10^{-14}$ to select the spectra of objects of the desired class. The ratio (17) for objects that do not belong to the class is equal up to one.

6. CONCLUSIONS

The presented study shows that the use of a signal with regression properties, that is, having a zero mean, as a response signal of correlation filters, makes it possible to improve the quality of recognition of a given class of objects.

The linearization of the ridge regression optimization problem is proposed by using the SVD of input data matrix.

A scheme for determining the optimal value of the regularization parameter is proposed.

The capture bandwidth of objects recognition and errors rate of the inverse filters depend on the level of the regularizatio parameter. Therefore, choosing a method for searching for the optimal regularization parameter is important for obtaining high quality object recognition by a bank of ICF.

The choice of regularization also affects the dynamics of the filters training process. The process can be with monotonically improving characteristics, or with the achievement of a certain maximum.

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