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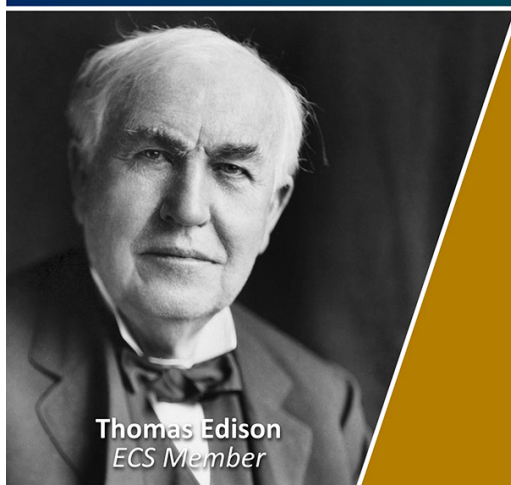
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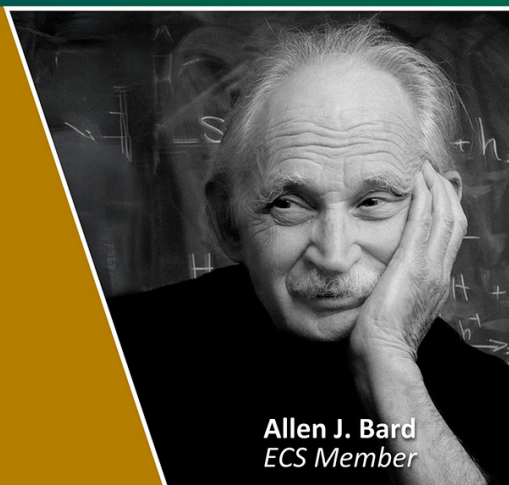


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# Increase in the energy efficiency of the biogas production process through vibration activation of heat and mass exchange processes in the bioreactor

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**Abstract.** One of the important problems preventing the increase in energy efficiency of biogas reactors is the use of heating elements operating in the free-convective mode of heat exchange. In addition to the large heat exchange areas, and the correspondingly reduced useful volume of the reactor, the use of free convective heat exchangers leads to the formation of stagnation zones, stratification of the medium, hypocoooling of the lower zones of the reactor, overheating of the upper zones. The intensification of the heat exchange between the heating element and the substrate will allow to reduce the area of the heat exchange devices in the reactor, activate the thermostabilizing processes and reduce the cost of the reactor construction. Based on the results of the analysis of the influence of the fluid oscillation created by the movement of the activator plate on the intensity of heat transfer from the heating wall to the water and the substrate, the choice of the determining temperature of the liquid and the temperature of the wall with different methods of organizing convection is substantiated.

## 1. Introduction

The profitability of biogas plants is inversely proportional to the energy consumption for heating the reactor. Microbiological features of the anaerobic fermentation process require compliance with the limits of temperature regimes and temperature stabilization, uniform heating of the environment, absence of hypothermic and overheating zones.

In paper [1] is investigated the phenomena of biogas production and mass transfer for individual anaerobic granules in a microreactor.

A thorough examination of biogas production performance in both submerged and external Anaerobic Membrane Bioreactors featuring polymeric and ceramic membranes is conducted by Tomczak et al. [2]. The study places particular emphasis on analyzing the influence of temperature and hydraulic retention time on biogas yield. Additionally, detailed descriptions of strategies to mitigate fouling are provided.

In [3] is formed a neural network model of the investment process of biogas production, which allows to increase the efficiency of the management decision-making process on the feasibility of investing in biogas plants.

Terziev, Zlateva and Ivanov [4] considers enhancing the fermentation process in the digester for biogas production, obtained from animal and plant waste substrates.

Pilarski et al. [5] analyze energy conversion in an active biogas plant as an open thermodynamic system.



Abanades et al. [6] provide a brief overview of some safety rules and proposed policies based on world regulations. The effect of these regulations and policies on marketing and promoting biogas is highlighted for different countries.

In the course of anaerobic fermentation of biomass, it is necessary to observe certain limits of temperature regimes [7–14]:

- psychrophilic – substrate temperature up to 20°C, fluctuations  $\pm 3...4^{\circ}\text{C}$ ;
- mesophilic – substrate temperature 35°C, fluctuations  $\pm 2.8^{\circ}\text{C}$ ;
- thermophilic – substrate temperature 55°C, fluctuations  $\pm 0.3^{\circ}\text{C}$ .

At the same time, the speed of movement of the substrate under the conditions of the optimal fermentation process should not exceed 0.7 m/s. The expansion of the development of biogas plants requires a reduction in their capital and operating costs.

One of the important problems preventing the increase in energy efficiency of biogas reactors is the use of heating elements operating in the free-convective mode of heat exchange. In addition to the large heat exchange areas, and the correspondingly reduced useful volume of the reactor, the use of free convective heat exchangers leads to the formation of stagnation zones, stratification of the medium, hypocooling of the lower zones of the reactor, overheating of the upper zones. The intensification of the heat exchange between the heating element and the substrate will allow to reduce the area of the heat exchange devices in the reactor, activate the thermostabilizing processes and reduce the cost of the reactor construction.

## 2. Results and discussion

In literary sources, the following methods of intensification of free-convective heat exchange under the conditions of external washing of the surface are of interest [7, 11–14]:

- 1) the formation of a forced flow near the heating surface due to the installation of vertical limiting walls;
- 2) increasing the heat exchange surface by finning, which can both increase and decrease the intensity of heat exchange;
- 3) local and massive bubbling of the heat exchange surface by biogas jets, while the intensity of heat exchange increases by 1.5...2.0 times;
- 4) vibration of the heat exchange surface, which increases the intensity of heat transfer for water up to 20 times, but it is undesirable due to structural complications of the heat supply and vibration drive systems;
- 5) vibration of the environment, acoustic methods, the maximum achievable intensification of heat exchange 2...4 times.

The method based on the vibration of the environment is the most suitable for the conditions of the biogas plant, since, in addition to the intensification of heat transfer, additional mixing takes place. Also, among the most effective methods of increasing the intensity of heat exchange when liquid flows around bodies is the vibration of the environment, which provides, under optimal conditions, an increase in the intensity of heat exchange by 2-4 times compared to free convection. The high intensity of heat exchange during the vibration of the medium and the relatively small consumption of power for its implementation, the simplicity and flexibility of managing this process – all this provides the advantages of vibration activation in relation to other ways of intensifying heat exchange in liquids and gases. These advantages have led to a fairly widespread of vibration activation in modern technology. Washing surfaces with liquid waves is used in power engineering, metallurgy, chemical technology, air conditioning systems, and other fields of technology.

Oscillation of the liquid with the help of a pulsator-valve, a crank mechanism, etc. allows to create of artificial turbulence in the area of the heat exchange surface, which will destroy the viscous wall layer, intensive washing of the heating wall and intensification of heat exchange. For small and medium-sized reactors, one of the effective mechanisms for stirring is the engine-driven crank-rod stirrer. Consider the movement of the crank mechanism, which was used as a pulsation exciter in the biogas reactor (figure 1). Let's assume that the mechanism activates the activator plate, which is placed above the heat exchanger and performs oscillating movements above it. The plate can be both solid and perforated, its shape determines the hydraulic resistance of the medium – the substrate.

The equation of motion of the activator plate can be found by solving the Lagrange equation:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\phi}} \right) - \frac{\partial T}{\partial \phi} = Q\phi, \quad (1)$$

The total kinetic energy of the system is defined as the sum of the kinetic energies of the movement of the components of the crank-connecting mechanism – the flywheel, the crank and the activator plate:

$$T = T_1 + T_2 + T_3, \quad (2)$$

To determine the kinetic energies of the components of the system, we write down the kinematic relations between the elements.

Having entered the coordinate system, as shown in figure 1, we write down the kinematic relations between the components of the activator mechanism and the velocity equation, provided that the instantaneous center of velocity is point P<sub>2</sub>.

$$KA = r \cdot \sin(\phi) - \epsilon, \quad (3)$$

$$KB = \sqrt{l^2 - (\sin(\phi) - \epsilon)^2}. \quad (4)$$

$$\left\{ \begin{array}{l} Y_B = r \cdot \cos(\phi) + KB \\ X_B = \epsilon \end{array} \right\} \quad (5)$$

$$\left\{ \begin{array}{l} X_c = \epsilon + \frac{KA}{2} = \epsilon + (r \cdot \sin(\phi) - \epsilon)/2 \\ Y_c = r \cdot \cos(\phi) + \frac{KB}{2} = r \cdot \cos(\phi) + 0.5 \cdot \sqrt{l^2 - (r \cdot \sin(\phi) - \epsilon)^2} \end{array} \right\} \quad (6)$$

$$\left\{ \begin{array}{l} V_B = \dot{Y}_B = \dot{\epsilon} * r * \sin(\phi) + \frac{-(r \cdot \sin(\phi) - \epsilon) \cdot r \cdot \cos(\phi) \cdot \dot{\phi}}{\sqrt{l^2 - (r \cdot \sin(\phi) - \epsilon)^2}} \\ V_C = \sqrt{\dot{X}_c^2 + \dot{Y}_c^2} = -\sqrt{\frac{r^2 \cdot \dot{\phi}^2 \cdot \cos^2(\phi)}{4} + \left| -r \cdot \dot{\phi} \cdot \sin(\phi) - \frac{\dot{\phi} \cdot r \cdot \cos(\phi) \cdot (r \cdot \sin(\phi) - \epsilon)^2}{\sqrt{l^2 - (r \cdot \sin(\phi) - \epsilon)^2}} \right|} \end{array} \right\} \quad (7)$$

The distance  $h_2$ , which will be covered by the center of mass of the link PK – point C, during the movement of the mechanism, is determined according to the equation:

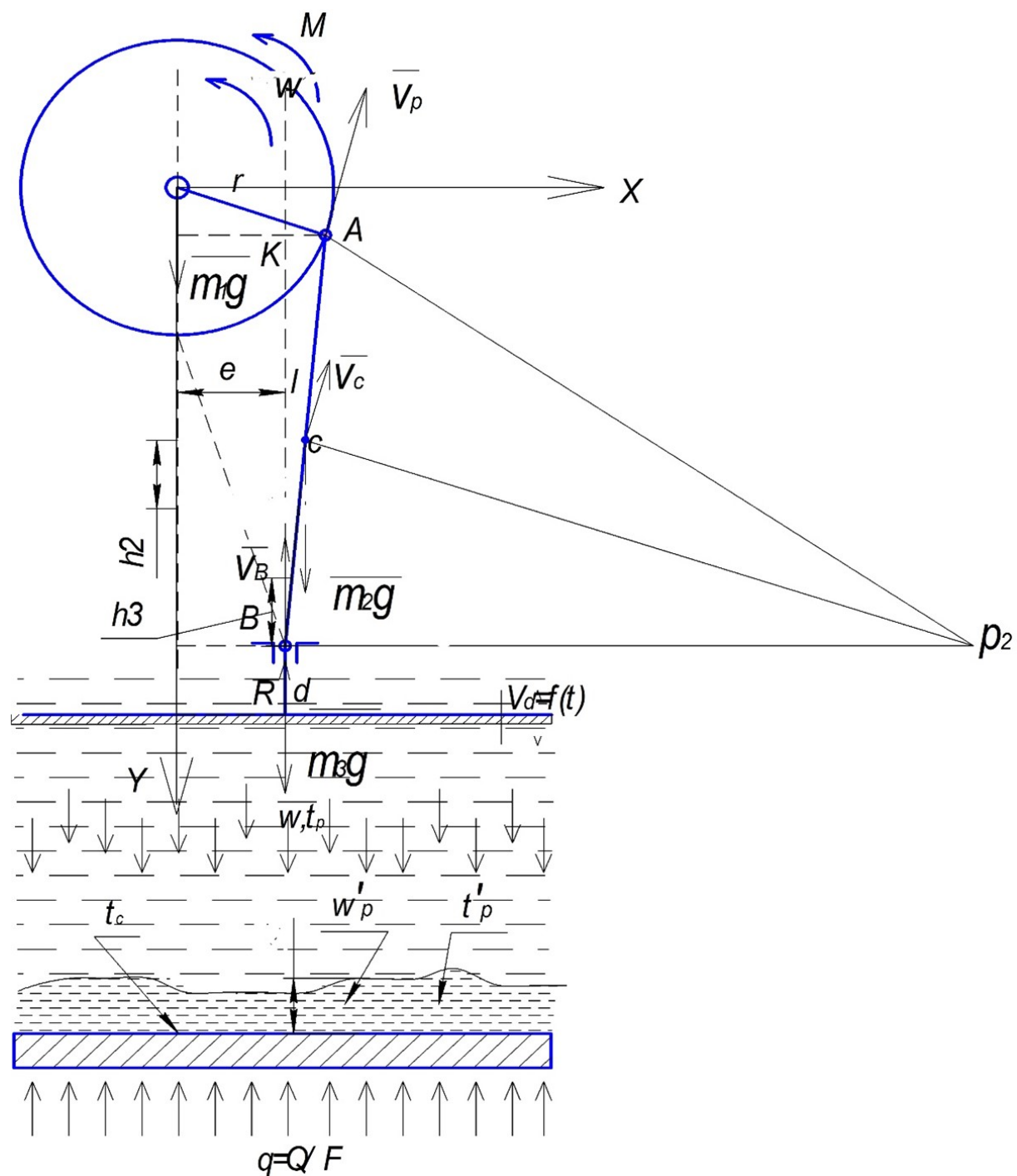
$$h_2 = r + \sqrt{l^2 - \epsilon^2}/2 - \left( r \cdot \cos(\phi) + \sqrt{l^2 - 0.5 \cdot (r \cdot \sin(\phi) - \epsilon)^2} \right), \quad (8)$$

The distance  $h_3$  passed by point B during movement of the mechanism is determined from the equation:

$$h_3 = r + \sqrt{l^2 - \epsilon^2} - \left( r \cdot \cos(\phi) + \sqrt{l^2 - (r \cdot \sin(\phi) - \epsilon)^2} \right), \quad (9)$$

Let's find the equation describing the value of the generalized force  $Q\phi$ :

$$Q\phi = \frac{\delta A}{\delta \phi}. \quad (10)$$



**Figure 1.** Dynamic diagram of the movement of the working organ of the bioreactor with an activator plate.

The total work of the forces acting on this activator mechanism consists of the work of the weight of the crank link –  $A_m 2g$  weight forces of the slider –  $A_m 3g$ ; work moment of forces, which sets the activator in motion –  $A_M$ ; the work of the resistance force of the movement medium of the activator plate is  $A_R$ .

$$A = A_m 2g + A_m 3g + A_M + A_R \quad (11)$$

Let's find the meaning of these works:

$$A_m 2g = -m_2 \cdot g \cdot \left[ r \cdot (1 - \cos(\phi)) + \frac{\sqrt{(l^2 - \epsilon^2)} - \sqrt{(l^2 - (r \cdot \sin(\phi) - \epsilon)^2)}}{2} \right] \quad (12)$$

$$A_m 3g = -m_2 \cdot g \cdot \left[ r \cdot (1 - \cos(\phi)) + \sqrt{(l^2 - \epsilon^2)} - \sqrt{(l^2 - (r \cdot \sin(\phi) - \epsilon)^2)} \right] \quad (13)$$

$$A_M = M \cdot \phi \quad (14)$$

We write down the resistance force of the movement medium of the activator plate in the form:

$$R = C \cdot S \cdot \frac{\rho \cdot Wmax^2}{2} \quad (15)$$

where  $C$  is the experimental coefficient [15], which depends on the aspect ratio of the plate;  $Wmax$  is the maximum permissible speed of movement of the substrate ( $Wmax = 0.7$  m/s). Using the value of the resistance of the medium in the form (15) allows us to take into account only the technologically maximum permissible value of the speed of movement of the substrate, thus simplifying the engineering solution of equation (1).

$$A_R = -0.245 \cdot m_3 \cdot \rho \cdot C \cdot S \cdot \left( r + \sqrt{(l^2 - \epsilon^2)} - (1 \cdot \cos(\phi) + \sqrt{(l^2 - (r \cdot \sin(\phi) - \epsilon)^2}) \right) \quad (16)$$

Then the expression for finding the generalized force will have the form:

$$\begin{aligned} Q = & M - m_2 \cdot g \cdot \left( r \cdot \sin(\phi) + \frac{\cos(\phi) \cdot (r \cdot \sin(\phi) - \epsilon)}{2 \cdot \sqrt{l^2 - (r \cdot \sin(\phi) - \epsilon)^2}} \right) - m_3 g \cdot \\ & - m_3 g \cdot \left[ r \cdot \sin(\phi) + \frac{r \cdot \cos(\phi) \cdot (r \cdot \sin(\phi) - \epsilon)}{2 \cdot \sqrt{l^2 - (r \cdot \sin(\phi) - \epsilon)^2}} \right] - \\ & - 0.245 \cdot m_3 \cdot \rho \cdot C \cdot S \cdot \left( r + \sqrt{(l^2 - \epsilon^2)} - (r \cdot \cos(\phi) + \sqrt{(l^2 - (r \cdot \sin(\phi) - \epsilon)^2}) \right) \end{aligned} \quad (17)$$

The kinetic energy of the system is the sum of the kinetic energies of the component links. Taking into account the peculiarities of the movement of each of the components, we will write down the equation for determining the kinetic energies of the links of the oscillatory activator:

$$T = T_1 + T_2 + T_3 \quad (18)$$

The kinetic energy of the first body is rotational motion:

$$T_1 = \frac{J \cdot \dot{\phi}^2}{2} = \frac{m_1 \cdot r^2 \cdot \dot{\phi}^2}{4} \quad (19)$$

The kinetic energy of the second body is plane-parallel motion:

$$\begin{aligned} T_2 = & \dot{\phi} \cdot r^2 \cdot m_2 \cdot 0.5 \left[ \left[ \frac{[r \cdot \cos(\phi) + \sqrt{(l^2 - (r \cdot \sin(\phi) - \epsilon)^2}] \cdot \tan(\phi) - \epsilon]^2}{l^2 - (r \cdot \sin(\phi) - \epsilon)^2} \right] + \right. \\ & + 0.25 \cdot l^2 - [r \cdot \cos(\phi) + \sqrt{(l^2 - (r \cdot \sin(\phi) - \epsilon)^2}] \cdot \tan(\phi) - \epsilon \cdot \\ & \left. \cdot (r \cdot \sin(\phi) - \epsilon) + \frac{l^2}{12 \cdot [l^2 - (r \cdot \sin(\phi) - \epsilon)^2]} \right] \end{aligned} \quad (20)$$

The kinetic energy of the third body is translational motion

$$T_3 = \phi^2 \cdot r^2 \cdot \left[ \frac{m_3 \left[ (r - \epsilon) \cos(\phi) + [l^2 - (r \cdot \sin(\phi) - \epsilon)^2]^2 \right]}{2 \cdot (\cos(\phi))^2 \cdot [l^2 - (r \cdot \sin(\phi) - \epsilon)^2]} \right] \quad (21)$$

The expression for determining the kinetic energy will look like this:

$$\begin{aligned} T = \phi^2 \cdot r^2 \cdot & \left[ 0.25m_1 + \frac{m_3 \left[ (r - \epsilon) \cos(\phi) + [l^2 - (r \cdot \sin(\phi) - \epsilon)^2] \right]^2}{2 \cdot (\cos(\phi))^2 \cdot [l^2 - (r \cdot \sin(\phi) - \epsilon)^2]} + \right. \\ & + m_2 \cdot 0.5 \left[ \frac{r \cdot \cos(\phi) + \sqrt{[l^2 - (r \cdot \sin(\phi) - \epsilon)^2]} \cdot \tan(\phi) - \epsilon^2}{(l^2 - (r \cdot \sin(\phi) - \epsilon)^2)} \right] + \\ & + 0.25 \cdot l^2 - \left[ r \cdot \cos(\phi) + \sqrt{[l^2 - (r \cdot \sin(\phi) - \epsilon)^2]} \cdot \tan(\phi) - \epsilon \right] \cdot \\ & \left. \cdot (r \cdot \sin(\phi) - \epsilon) + \frac{l^2}{12 \cdot [l^2 - (r \cdot \sin(\phi) - \epsilon)^2]} \right] \end{aligned} \quad (22)$$

The partial derivative of the kinetic energy along the angle of rotation:

$$\begin{aligned} \frac{\delta T}{\delta \phi} = & (m_3^2 \cdot \frac{[(r - e) \cdot \cos(\phi) + l^2 - (r \cdot \sin(\phi) - e)^2]}{\cos(\phi)^2 \cdot [l^2 - (r \cdot \sin(\phi) - e)^2]} \cdot \\ & \cdot [(-r - e) \cdot \sin(\phi) - 2(r \cdot \sin(\phi) - e) \cdot r \cdot \cos(\phi)] + \\ & + m_3^2 \cdot \frac{[(r - e) \cdot \cos(\phi) + l^2 - (r \cdot \sin(\phi) - e)^2]^2}{\cos(\phi)^3 \cdot [l^2 - (r \cdot \sin(\phi) - e)^2]} \cdot \sin(\phi) + \\ & + m_3^2 \cdot \frac{[(r - e) \cdot \cos(\phi) + l^2 - (r \cdot \sin(\phi) - e)^2]^2}{\cos(\phi) \cdot [l^2 - (r \cdot \sin(\phi) - e)^2]} \cdot (r \cdot \sin(\phi) - e) \cdot r + \\ & + 0.5 \cdot m_2 \cdot \left[ 2 \cdot \frac{[r \cdot \cos(\phi) + [l^2 - (r \cdot \sin(\phi) - e)^2]^{1/2} \cdot \tan(\phi) - e]}{l^2 - (r \cdot \sin(\phi) - e)^2} \cdot \right. \\ & \cdot [-r \cdot \sin(\phi) - \frac{1}{[l^2 - (r \cdot \sin(\phi) - e)^2]^{1/2}} \cdot \tan(\phi) \cdot (r \cdot \sin(\phi) - e) \cdot \\ & \cdot r \cdot \cos(\phi) + [l^2 - (r \cdot \sin(\phi) - e)^2]^{1/2} \cdot (1 + \tan(\phi)^2) + \\ & + 2 \cdot \frac{[r \cdot \cos(\phi) + [l^2 - (r \cdot \sin(\phi) - e)^2]^{1/2} \cdot \tan(\phi) - e]^2}{l^2 - (r \cdot \sin(\phi) - e)^2} \cdot (r \cdot \sin(\phi) - e) \cdot r \cdot \cos(\phi) - \\ & - [-r \cdot \sin(\phi) - \frac{1}{[l^2 - (r \cdot \sin(\phi) - e)^2]^{1/2}} \cdot \tan(\phi) \cdot (r \cdot \sin(\phi) - e) \cdot \\ & \cdot r \cdot \cos(\phi) + [l^2 - (r \cdot \sin(\phi) - e)^2]^{1/2} \cdot (1 + \tan(\phi)^2) \cdot (r \cdot \sin(\phi) - e) - \\ & - [r \cdot \cos(\phi) + [l^2 - (r \cdot \sin(\phi) - e)^2]^{1/2} \cdot \tan(\phi) - e] \cdot r \cdot \cos(\phi) + \\ & \left. \left. + \frac{l^2}{[12l^2 - 12(r \cdot \sin(\phi) - e)^2]^2} \cdot (r \cdot \sin(\phi) - e) \cdot r \cdot \cos(\phi) \right] \right] \cdot \phi^2 \cdot r^2 \end{aligned} \quad (23)$$

In (22), we denote the expression in parentheses by “ $D$ ”, then the time derivative:

$$\frac{d}{dt} \left( \frac{\delta T}{\delta \dot{\phi}} \right) = 2 \cdot \ddot{\phi} \cdot r^2 \cdot D \quad (24)$$

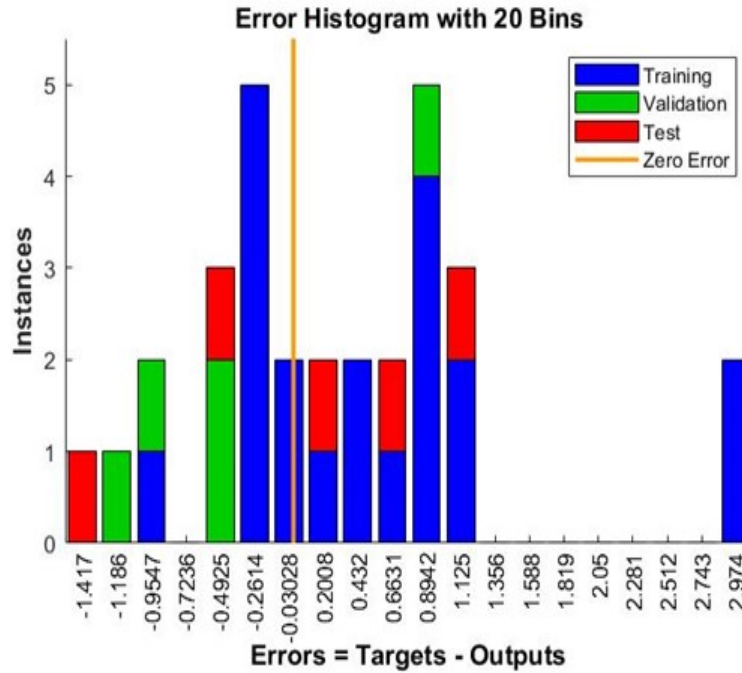
The Lagrange equation (1) will be written in the form:

$$2 \cdot \ddot{\phi} \cdot r^2 \cdot D - \frac{\delta T}{\delta \phi} = Q_\phi, \quad (25)$$

where  $\delta T/\delta \phi$  and  $Q_\phi$  are determined from equations (23) and (17), respectively. Let's turn the resulting nonlinear differential equation into a system of linear differential equations of the first order by substitution:

$$\begin{bmatrix} \ddot{\phi} = \dot{z} \\ \dot{\phi} = z \end{bmatrix} \quad (26)$$

We will solve this system using the Runge–Kutta numerical method and obtain a solution in the form of values of the function  $\omega = f(\varphi)$ . Using the solution of system (26), it is possible to determine the value of the speed of the activator plate. The liquid under the action of the plate also begins to make oscillatory movements and washes the heat exchange surface. A wall layer is located around the heat exchange surface, in which heat is transferred mainly due to thermal conductivity (figure 2).



**Figure 2.** Thermal and hydrodynamic situations around the heat exchange surface according to the hypothesis of Reynolds and Prandtl.

According to the Fourier equation, we write that the heat flow  $q$  is determined:

$$q = \lambda \cdot \frac{t'_p - t_c}{\delta'_t} \quad (27)$$

where  $\delta'_t$  is the thickness of the thermal layer;  $t'_p$  is the temperature of the liquid in the wall layer.

According to Kays, Crawford and Weigand [16], and Bergman et al. [17] the thickness of the viscous wall hydrodynamic  $\delta'$  and thermal  $\delta'_t$  layers is determined from the dependencies:

$$\delta' \approx 12 \frac{\nu}{\sqrt{s/\rho}} \quad (28)$$

$$\delta'_t \approx 12 \frac{\nu Pr^{-0.33}}{\sqrt{s/\rho}} \quad (29)$$

The thermophysical properties of the substrate are determined from equations [7]:

$$\rho = 1000 + 2.4 \cdot c; \quad (30)$$

$$c_p = c_{dry} \cdot c + c_w \cdot (1 - c); \quad (31)$$

$$\mu = \mu_w \cdot \left[ 1 + 10 \cdot \left( \frac{10 \cdot (c - 2)}{(11 - \beta_u \cdot c)} + c \right) \right], \quad (32)$$

where  $c$  is the content of dry matter, 2–15%;  $c_{dry}$  – heat capacity of the dry substance of the substrate;  $c_w$  – heat capacity of water;  $\mu_w$  is the viscosity of water at a certain temperature;  $\beta_u$  is a coefficient that depends on the type of waste and is 0.7 ... 0.75 for cattle and  $\beta_u = 0.6 \dots 0.7$  for pigs.

According to Newton's equation, the value of the tangential stress is defined as:

$$s = \mu_w \frac{w'}{\delta'} = \mu_w \cdot \left[ 1 + 10 \cdot \left( \frac{10 \cdot (c - 2)}{(11 - \beta_u \cdot c)} + c \right) \right] \cdot \frac{w'}{\delta'}, \quad (33)$$

where  $w'$  is the speed of movement of substrate particles that have entered the wall layer.

According to the Reynolds theory, supplemented by Prandtl hypothesis, the fluid flow washing over the heat exchange surface and having an average speed  $w$ , falling into the wall layer, slows down and acquires a speed  $w'$ . The temperature of the liquid inside the flow is  $t_p$ , the temperature of the liquid inside the wall layer is  $t'_p$ . According to the law of conservation of momentum, the force of motion resistance will have the form:

$$S = G' \cdot (w - w'), \quad (34)$$

where  $G'$  is the amount of liquid that entered the wall layer per unit of time. The amount of heat transferred by the liquid  $G'$  will be determined according to the expression:

$$Q = G' \cdot c_p \cdot (t_p - t'_p) \quad (35)$$

Then, given that  $S = s \cdot F$  and  $Q = q \cdot F$ , we get:

$$q = c_p \cdot s \cdot \frac{t_p - t'_p}{w - w'} \quad (36)$$

From equations (29) and (33), the velocity in the wall layer  $w'$  is determined:

$$w' = 12 \sqrt{s/\rho} \quad (37)$$

Temperature gradients are determined from equations (27) and (35).

$$t'_p - t_c = q \cdot \delta'_t / \lambda \quad (38)$$

$$t_p - t'_p = \frac{q \cdot (w - w')}{c_p \cdot s} \quad (39)$$

Adding expressions (38) and (39), we get:

$$t_p - t_c = \frac{q \cdot (w - w')}{c_p \cdot s} + q \cdot \delta'_t / \lambda \quad (40)$$

Let's transform equation (40), taking into account that

$$\alpha = \frac{q}{t_c - t_p}$$

$$\alpha = \left[ \frac{w - 12\sqrt{s/\rho}}{c_p \cdot s} + \frac{12v}{\lambda\sqrt{s/\rho}} \right]^{-1} \quad (41)$$

The values of isobaric heat capacity, density, and viscosity can be determined using equations (30), (31), (32).

The speed of fluid movement  $w$  is determined from the solution of system (26), assuming that the fluid moves to the heat exchange surface at the speed of movement of the activator plate. In the criterion form, equation is written as:

$$Nu = \frac{d}{\lambda} \left[ \frac{w - 12\sqrt{s/\rho}}{c_p \cdot s} + \frac{12v}{\lambda\sqrt{s/\rho}} \right]^{-1} \quad (42)$$

The value of the tangential stress  $s$  during vibration washing of a single cylindrical surface is currently unknown. In the first approximation, it is possible to take the value of the tangential stress determined for the turbulent washing of the plate, transforming it as follows:

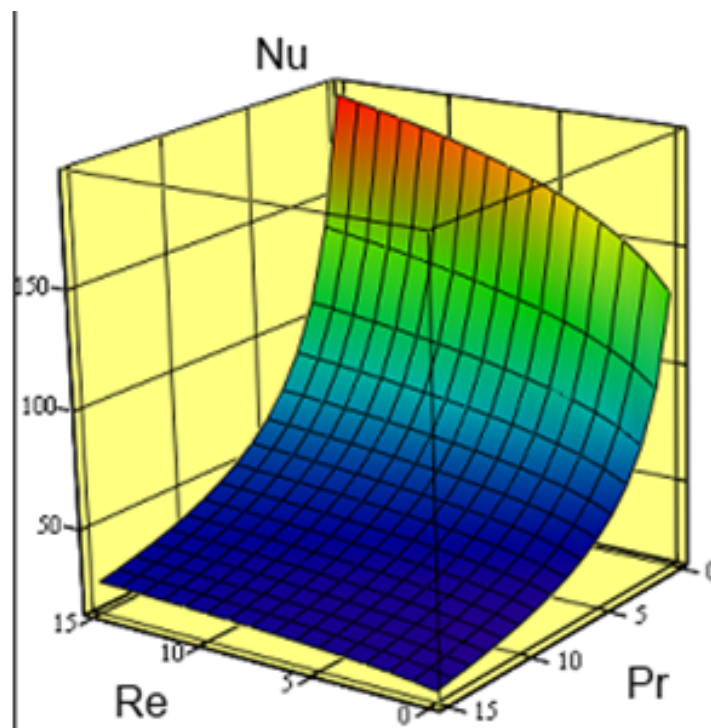
$$s = 0.037 \cdot \rho \cdot \frac{(w_A)^2}{2} \cdot (w_a \cdot d/2 \cdot v)^{-0.2} \quad (43)$$

Thus, equation (42) makes it possible to determine the value of the Nusselt criterion during oscillatory washing of the heat-exchange cylindrical surface in bioreactors for the utilization of organic waste of agricultural enterprises.

Based on the results of the analysis of the developed mathematical model, it has been found that in the range of measurements, the obtained value of the increase in the coefficient of heat transfer from the horizontal heating tubular element to the water exceeds the value for free convection by an average of 1.5...2.8 times. Both medium fluctuations and heat flow have a significant influence on the intensification of heat exchange. Moreover, an increase in the frequency of oscillations affects the increase in the coefficient of heat transfer from the horizontal cylinder to the water more significantly than an increase in the amplitude by an average of 15–25%. After analyzing equation (43), the surface of the influence of the arguments included in this equation –  $Re$  and  $Pr$  – on the value of the function – the Nusselt criterion (figure 3) was constructed.

The proposed mathematical model calculates the heat exchange processes in biogas reactors under vibration activation of the substrate heating process from the tubular heat exchange element. Based on the parameters of the heat exchange subsystem of the reactor: the heat exchange area, the frequency and amplitude of oscillations, the temperature of the coolant, the thermophysical properties of the substrate, choosing one of the three fermentation temperature regimes, it is possible to determine the heat exchange coefficient from the surface of the heater to the substrate, calculate the amount of heat transferred, carry out design calculations of the heat exchange process. In addition to activating heat exchange processes in the substrate environment, temperature fields are equalized, and warming is more uniform.

Due to the activation of the heat exchange, a reduction in the mass of the heat exchange process for the  $V = 3 \text{ m}^3$  reactor was achieved by 18% a decrease in the cost of the reactor by 7% due to a lower cost of the heat exchange process (while taking into account the increase in the price of the reactor due to the vibration activator), an increase in the yield of biogas due to thermal stabilization by 12...14% compared to the free convective heating process. The results were obtained at a model reactor and as a result of implementation in a small farm in the Vinnytsia region.



**Figure 3.** The surface of the influence of the Reynolds and Prandtl criteria on the value of the Nusselt criterion during vibration activation of heat exchange.

### 3. Conclusions

The law of motion of the activator plate has been found by solving Lagrange's differential equation. Based on the laws of thermal conductivity, convective heat and mass transfer, and conservation of momentum, a criterion equation has been developed that describes the heat exchange processes between the heating element and the substrate during oscillatory washing of the heater wall.

The mathematical model was checked for adequacy by comparing the obtained results with the results of experimental studies. For this, several experimental data were collected. Based on the comparison of two samples using the Student's criterion, the model was evaluated, errors were calculated, and a t-test was performed. It was determined that the t value does not exceed the critical value for the given degrees of freedom and the significance level of 0.05. That is, the developed mathematical model is adequate.

Based on the results of the analysis of the influence of the fluid oscillation created by the movement of the activator plate on the intensity of heat transfer from the heating wall to the water and the substrate, the choice of the determining temperature of the liquid and the temperature of the wall with different methods of organizing convection is substantiated. It has been found that the oscillation of the medium by the activator plate intensifies the free convective heat transfer in the aqueous medium by 1.5...3.0 times, in the  $C = 8\%$  suspension by 1.3...2.5 times. Thus, the use of an activator plate to intensify heat exchange in bioreactors allows thermostabilization and intensification of the biogas production process and saves material resources by reducing the area of heat exchange devices.

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